

Supplementary Material

We provide additional theoretical and experimental details that complement the main paper. In Section A, we present full proofs of Theorem 2.1. Section B explains the sample complexity analysis that underlies the bounds for T-AAB reported in Table 1. Sections C, D provide additional experimental results: 3D reconstructions (Section C and D), supplementary figures to the ETH3D dataset (Section E), runtime comparisons (Section F), and extended results on rotation synchronization (Section G), and generalization to the IMC-PT dataset (Section H).

All equation, figure, table, and theorem numbers continue from the main paper.

A Proofs of theory

We first establish Theorem A.1 and then conclude the main theorem.

Theorem A.1. Assume there exists an absolute $\alpha > 0$ such that for any $ij \in E_g$ and $k \in N_{ij}$, $\alpha < \theta_{ij,k} < \pi - \alpha$. Then for $ij \in E_g$, we have $\tilde{d}_{ij,k} \leq C_\alpha(\tilde{s}_{ik}^* + \tilde{s}_{jk}^*)$, where $C_\alpha = \frac{2(\cos \alpha + \sqrt{5-4\cos^2 \alpha})}{\sin^2 \alpha}$.

We note that the triangle is ill-shaped whenever $\theta_{ij,k} \approx 0$ or $\theta_{ij,k} \approx \pi$. In practice, we want $d_{ij,k}$ to be small for a clean edge $ij \in E_g$ whenever the other two edges are relatively clean with small \tilde{s}_{ik}^* and \tilde{s}_{jk}^* . However, in these two ill-shaped cases, C_α in the theorem goes to infinity, and there is no effective upper bound to control $\tilde{d}_{ij,k}$.

Proof. Let γ_p be the projected vector of γ_{ij} onto $\text{Span}(\gamma_{ik}, \gamma_{jk})$. Denote $x = \gamma_{ij}^T \gamma_{ki}$, $y = \gamma_{ij}^T \gamma_{jk}$ and $z = \gamma_{jk}^T \gamma_{ki}$. Since γ_p is in $\text{Span}(\gamma_{ki}, \gamma_{jk})$, there exists constants a, b such that $\gamma_p = a\gamma_{ki} + b\gamma_{jk}$. By the definition of γ_p , we have

$$\begin{cases} \langle \gamma_{ij} - a\gamma_{ki} - b\gamma_{jk}, \gamma_{ki} \rangle = 0 \\ \langle \gamma_{ij} - a\gamma_{ki} - b\gamma_{jk}, \gamma_{jk} \rangle = 0 \end{cases} \quad (11)$$

By linearity of vector inner products, we have

$$\begin{cases} x - a - bz = 0 \\ y - az - b = 0 \end{cases} \quad (12)$$

Solving for a and b gives

$$\begin{cases} a = \frac{x-yz}{1-z^2} \\ b = \frac{y-xz}{1-z^2} \end{cases} \quad (13)$$

Therefore $\gamma_p = \frac{x-yz}{1-z^2} \gamma_{ki} + \frac{y-xz}{1-z^2} \gamma_{jk}$.

Case 1: $\gamma_p \notin \Omega(\gamma_{jk}, \gamma_{ki})$. In this case, since $\tilde{d}_{ij,k} \leq 1$, we only need to prove $\tilde{s}_{ik}^* + \tilde{s}_{jk}^* > 1/C_\alpha$.

Note that by the definition of $\Omega(\gamma_{jk}, \gamma_{ki})$ we have either $a < 0$ or $b < 0$, which implies $x - yz > 0$ or $y - xz > 0$. On the other hand, since the ground truth directions γ_{ij}^* , γ_{jk}^* , γ_{ki}^* are cycle consistent, we know that the projection of γ_{ij}^* onto $\text{Span}(\gamma_{ki}^*, \gamma_{jk}^*)$ is in the set $\Omega(\gamma_{ki}^*, \gamma_{jk}^*)$. Therefore we also have $x^* - y^*z^* < 0$ and $y^* - x^*z^* < 0$, where $x^* = \gamma_{ij}^{*T} \gamma_{ki}^*$, $y^* = \gamma_{ij}^{*T} \gamma_{jk}^*$ and $z^* = \gamma_{jk}^{*T} \gamma_{ki}^*$. Without loss of generality, we assume the case $x - yz > 0$. If $\max(\tilde{s}_{ik}^*, \tilde{s}_{jk}^*) \geq \frac{1}{C_\alpha}$, then the lemma is trivial. If $\max(\tilde{s}_{ik}^*, \tilde{s}_{jk}^*) < \frac{1}{C_\alpha}$, we first verify two claims.

Claim 1: $x^* - y^*z^* < -\sin \alpha^*$, where α^* is the angle such that $\cos \alpha^* = \cos \alpha + \frac{2}{C_\alpha}$.

By the definition of \tilde{s}_{ik}^* and \tilde{s}_{jk}^* , we have the following inequality:

$$|\gamma_{ik} - \gamma_{ik}^*| + |\gamma_{jk} - \gamma_{jk}^*| = 2 \sin \frac{\tilde{s}_{ik}^*}{2} + 2 \sin \frac{\tilde{s}_{jk}^*}{2} \leq \tilde{s}_{ik}^* + \tilde{s}_{jk}^* \leq \frac{2}{C_\alpha}. \quad (14)$$

Also, $\alpha < \theta_{ij,k} < \pi - \alpha$ is equivalent to $|\gamma_{jk}^T \gamma_{ki}| = |\cos \theta_{ij,k}| < \cos \alpha$. Combining this with equation (14) gives

$$\begin{aligned}
|\gamma_{jk}^{*T} \gamma_{ki}^*| &= |\gamma_{jk}^T \gamma_{ki} + (\gamma_{jk}^* - \gamma_{jk})^T \gamma_{ki} + \gamma_{jk}^{*T} (\gamma_{ki}^* - \gamma_{ki})| \\
&\leq |\gamma_{jk}^T \gamma_{ki}| + |(\gamma_{jk}^* - \gamma_{jk})^T \gamma_{ki}| + |\gamma_{jk}^{*T} (\gamma_{ki}^* - \gamma_{ki})| \\
&\leq |\gamma_{jk}^T \gamma_{ki}| + |\gamma_{jk}^* - \gamma_{jk}| + |\gamma_{ki}^* - \gamma_{ki}| \\
&\leq \cos \alpha + \frac{2}{C_\alpha} = \cos \alpha^*.
\end{aligned} \tag{15}$$

On the other hand, we know that $a^* = \frac{x^* - y^* z^*}{1 - z^{*2}}$, and $a^* = -|a^*| \leq -\sin \alpha^*$. This implies that $x^* - y^* z^* < -\frac{\sin \alpha^*}{1} = -\sin \alpha^*$.

Claim 2: Let $\delta_{ij} = \gamma_{ij} - \gamma_{ij}^*$, $\delta_{jk} = \gamma_{jk} - \gamma_{jk}^*$ and $\delta_{ki} = \gamma_{ki} - \gamma_{ki}^*$. Suppose $\max(|\delta_{ij}|, |\delta_{jk}|, |\delta_{ki}|) = \delta$. Then $|(x^* - y^* z^*) - (x - yz)| \leq 6\delta$; if $ij \in E_g$ (i.e. $\delta_{ij} = 0$), then $|(x^* - y^* z^*) - (x - yz)| \leq 4\delta$.

In fact, by the definition of x, x^*, y, y^*, z, z^* , we have the following estimate:

$$\begin{aligned}
|(x^* - y^* z^*) - (x - yz)| &= |(\gamma_{ij}^{*T} \gamma_{ki}^* - \gamma_{ij}^{*T} \gamma_{jk}^* \gamma_{jk}^{*T} \gamma_{ki}^*) - ((\gamma_{ij}^* + \delta_{ij})^T (\gamma_{ki}^* + \delta_{ki}) \\
&\quad - (\gamma_{ij}^* + \delta_{ij})^T (\gamma_{jk}^* + \delta_{jk}) (\gamma_{jk}^* + \delta_{jk})^T (\gamma_{ki}^* + \delta_{ki}))| \\
&\leq |-\delta_{ij}^T \gamma_{ki} - \gamma_{ij}^{*T} \delta_{ki} - (\delta_{ij}^T \gamma_{jk} \gamma_{jk}^T \gamma_{ki} + \gamma_{ij}^{*T} \delta_{jk} \gamma_{jk}^T \gamma_{ki} \\
&\quad + \gamma_{ij}^{*T} \gamma_{jk} \delta_{jk}^T \gamma_{ki} + \gamma_{ij}^{*T} \gamma_{jk} \gamma_{jk}^T \delta_{ki})| \\
&\leq |\delta_{ij}^T \gamma_{ki}| + |\gamma_{ij}^{*T} \delta_{ki}| + |\delta_{ij}^T \gamma_{jk} \gamma_{jk}^T \gamma_{ki}| + |\gamma_{ij}^{*T} \delta_{jk} \gamma_{jk}^T \gamma_{ki}| \\
&\quad + |\gamma_{ij}^{*T} \gamma_{jk} \delta_{jk}^T \gamma_{ki}| + |\gamma_{ij}^{*T} \gamma_{jk} \gamma_{jk}^T \delta_{ki}|.
\end{aligned} \tag{16}$$

By the fact that all γ 's are unit vectors, the right hand side of the equation above is at most 6δ in general; if $ij \in E_g$ (i.e. $\delta_{ij} = 0$) then it is at most 4δ .

Combining claim 1 and claim 2, we know that $0 < x - yz \leq (x^* - y^* z^*) + |(x - yz) - (x^* - y^* z^*)| \leq 4\delta - \sin \alpha^*$. This yields $\delta > \frac{\sin \alpha^*}{4}$. Note that by $ij \in E_g$, we know that $\delta_{ij} = 0$. Therefore $\delta = \max(|\delta_{ij}|, |\delta_{jk}|, |\delta_{ki}|) = \max(|\delta_{jk}|, |\delta_{ki}|) \leq |\delta_{jk}| + |\delta_{ki}| = 2 \sin \frac{\tilde{s}_{jk}^*}{2} + 2 \sin \frac{s_{ki}^*}{2} \leq \tilde{s}_{jk}^* + s_{ki}^*$. By $C_\alpha = \frac{2(\cos \alpha + \sqrt{5-4\cos^2 \alpha})}{\sin^2 \alpha}$, we know that $\frac{\sin \alpha^*}{4} = \frac{1}{C_\alpha}$, therefore the theorem is proved.

Case 2: $\gamma_p \in \Omega(\gamma_{ik}, \gamma_{kj})$. In this case, let $\delta_{ik} = \gamma_{ik} - \gamma_{ik}^*$ and $\delta_{jk} = \gamma_{jk} - \gamma_{jk}^*$. Then $\tilde{d}_{ij,k} = \frac{|\gamma_{ik} \times \gamma_{kj} \cdot \gamma_{ij}|}{\sin \theta_{ijk}}$. By the fact that $\gamma_{ij}^*, \gamma_{jk}^*, \gamma_{ki}^*$ are coplanar and $\gamma_{ij} = \gamma_{ij}^*$, we know that $\gamma_{ik}^* \times \gamma_{kj}^* \cdot \gamma_{ij} = 0$. We have the following inequalities:

$$\begin{aligned}
\tilde{d}_{ij,k} &= \frac{|\gamma_{ik} \times \gamma_{kj} \cdot \gamma_{ij}|}{\sin \theta_{ij,k}} \\
&= \frac{|(\gamma_{ik}^* + \delta_{ik}) \times (\gamma_{kj}^* + \delta_{kj}) \cdot \gamma_{ij}|}{\sin \theta_{ij,k}} \\
&= \frac{|(\delta_{ik} \times (\gamma_{kj}^* + \delta_{kj}) + \gamma_{ik}^* \times \delta_{kj}) \cdot \gamma_{ij}|}{\sin \theta_{ij,k}} \\
&\leq \frac{|\delta_{ik}| |\gamma_{kj}^* + \delta_{kj}| + |\gamma_{ik}^*| |\delta_{kj}|}{\sin \theta_{ij,k}} \\
&\leq \frac{\delta_{ik} + \delta_{kj}}{\sin \alpha}.
\end{aligned} \tag{17}$$

$$\leq \frac{\delta_{ik} + \delta_{kj}}{\sin \alpha}. \tag{18}$$

Note that $|\delta_{ik}| = 2 \sin \frac{\tilde{s}_{ik}^*}{2} \leq \tilde{s}_{ik}^*$, and similarly $|\delta_{jk}| = 2 \sin \frac{\tilde{s}_{jk}^*}{2} \leq \tilde{s}_{jk}^*$. Therefore $\tilde{d}_{ij,k} \leq \frac{1}{\sin \alpha} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*) \leq C_\alpha (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*)$, where the latter inequality comes from the fact that $C_\alpha \geq \frac{1}{\sin \alpha}$.

Proof of the main theorem.

Recall the statement of Theorem [2.1](#):

Main Theorem (Theorem [2.1](#)). Assume there exists $\alpha > 0$ such that for all $ij \in E_g$ and $k \in N_{ij}$, $\alpha < \theta_{ij,k} < \pi - \alpha$, and $\lambda < 1 + eC_\alpha/\mu - \sqrt{eC_\alpha(2\mu + eC_\alpha)}/\mu$, where $C_\alpha = 2(\cos \alpha + \sqrt{5 - 4\cos^2 \alpha})/\sin^2 \alpha$. Then, for $\tilde{s}_{ij,t}$ computed by the iteratively reweighted AAB algorithm [\[25\]](#) using $\beta_0 \leq \frac{1}{2\lambda}$ and $\beta_{t+1} = r\beta_t$ with $1 < r < \mu(1 - \lambda)^2/(2eC_\alpha\lambda)$, it holds for all $t > 0$ that

$$\forall ij \in E_g, \quad \tilde{s}_{ij,t} \leq \frac{1}{2\beta_0 r^t}, \quad \forall ij \in E_b, \quad \tilde{s}_{ij,t} \geq \frac{\mu}{e}(1 - \lambda)\tilde{s}_{ij}^*.$$

Proof. We prove the main theorem by induction. For $t = 0$, the definition of λ imply that for all $ij \in E$,

$$\tilde{s}_{ij}^{(0)} = \frac{\sum_{k \in N_{ij}} \tilde{d}_{ij,k}}{|N_{ij}|} \geq \frac{\sum_{k \in G_{ij}} \tilde{d}_{ij,k}}{|N_{ij}|} \geq \mu \frac{|G_{ij}|}{|N_{ij}|} \tilde{s}_{ij}^* \geq \mu(1 - \lambda)\tilde{s}_{ij}^*.$$

Furthermore, by the fact that $0 \leq \tilde{d}_{ij,k} \leq 1$ we have for all $ij \in E_g$,

$$\tilde{s}_{ij}^{(0)} = \frac{\sum_{k \in N_{ij}} \tilde{d}_{ij,k}}{|N_{ij}|} = \frac{\sum_{k \in B_{ij}} \tilde{d}_{ij,k}}{|N_{ij}|} \leq \frac{\sum_{k \in B_{ij}} 1}{|N_{ij}|} \leq \lambda \leq \frac{1}{2\beta_0}.$$

Therefore the theorem is proved when $t = 0$.

Next, we assume the theorem holds true for $0, 1, \dots, t$, and show that it also holds true for $t + 1$. By the definition of $\tilde{s}_{ij}^{(t+1)}$ and the induction assumption $\frac{1}{2\beta_t} \geq \max_{ij \in E_g} \tilde{s}_{ij,t}$, we have the following inequalities for any $ij \in E_b$:

$$\begin{aligned} \tilde{s}_{ij}^{(t+1)} &= \frac{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \tilde{d}_{ij,k}}{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})}} \\ &\geq \frac{\sum_{k \in G_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \tilde{d}_{ij,k}}{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})}} \\ &\geq \frac{\sum_{k \in G_{ij}} e^{-1} \tilde{d}_{ij,k}}{|N_{ij}|} \\ &\geq \frac{\mu}{e} \frac{|G_{ij}|}{|N_{ij}|} \tilde{s}_{ij}^* \\ &\geq \frac{\mu(1 - \lambda)}{e} \tilde{s}_{ij}^*. \end{aligned} \tag{19}$$

Next we bound $\tilde{s}_{ij}^{(t+1)}$ for $ij \in E_g$. By the definition of $\tilde{s}_{ij}^{(t+1)}$, the fact that $\tilde{d}_{ij,k} = 0$ for $k \in G_{ij}$, and Theorem [A.1](#) we know that

$$\tilde{s}_{ij}^{(t+1)} = \frac{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \tilde{d}_{ij,k}}{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})}} = \frac{\sum_{k \in B_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \tilde{d}_{ij,k}}{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})}} \tag{20}$$

$$\leq \frac{C_\alpha \sum_{k \in B_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*)}{\sum_{k \in N_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})}}. \tag{21}$$

By the induction assumption that $\tilde{s}_{ij}^{(t)} \geq \frac{\mu(1 - \lambda)}{e} \tilde{s}_{ij}^*$ for all $ij \in E$, we know that

$$\sum_{k \in B_{ij}} e^{-\beta_t(\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*) \leq \sum_{k \in B_{ij}} e^{-\beta_t \frac{\mu(1 - \lambda)}{e} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*)} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*). \tag{22}$$

Note that $xe^{-cx} < \frac{1}{ce}$ for any $c > 0$ and $x > 0$. Let $c = \beta_t \frac{\mu(1-\lambda)}{e}$ and $x = \tilde{s}_{ik}^* + \tilde{s}_{jk}^*$, we have

$$\sum_{k \in B_{ij}} e^{-\beta_t \frac{\mu(1-\lambda)}{e} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*)} (\tilde{s}_{ik}^* + \tilde{s}_{jk}^*) \leq \sum_{k \in B_{ij}} \frac{1}{\beta_t \mu(1-\lambda)} = \frac{|B_{ij}|}{\beta_t \mu(1-\lambda)}. \quad (23)$$

Also, by the induction assumption that $\frac{1}{2\beta_t} \geq \max_{ij \in E_g} \tilde{s}_{ij}^{(t)}$ and the nonnegativity of $\tilde{s}_{ij}^{(t)}$'s, we have

$$\sum_{k \in N_{ij}} e^{-\beta_t (\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \geq \sum_{k \in G_{ij}} e^{-\beta_t (\tilde{s}_{ik}^{(t)} + \tilde{s}_{jk}^{(t)})} \geq |G_{ij}| e^{-1}. \quad (24)$$

Combining (20), (22), (23), (24) and the definition of λ , we have

$$\tilde{s}_{ij}^{(t+1)} \leq \frac{|B_{ij}|}{|G_{ij}|} \cdot \frac{eC_\alpha}{\beta_t \mu(1-\lambda)} \leq \frac{2eC_\alpha \lambda}{\mu(1-\lambda)^2} \cdot \frac{1}{2\beta_t}. \quad (25)$$

By the assumption that $\lambda < 1 + \frac{eC_\alpha}{\mu} - \sqrt{\frac{eC_\alpha}{\mu}(2 + \frac{eC_\alpha}{\mu})}$, we know that $\frac{2e\lambda}{\mu(1-\lambda)^2} < 1$. Therefore by taking $1 < r < \frac{\mu(1-\lambda)^2}{2e\lambda}$, we guarantee that for any $ij \in E_g$, $\tilde{s}_{ij}^{(t+1)} \leq \frac{1}{2\beta_{t+1}} = \frac{1}{2\beta_0 r^{t+1}}$. This and (19) concludes our theorem. \square

Comment on the order of μ : We remark that in the theorem $\mu = \min_{ij \in E_b} \sum_{k \in G_{ij}} \tilde{d}_{ij,k} / (|G_{ij}| \tilde{s}_{ij}^*)$, which implies that for all $ij \in E$,

$$\frac{1}{|G_{ij}|} \sum_{k \in G_{ij}} \tilde{d}_{ij,k} \geq \mu \tilde{s}_{ij}^*. \quad (26)$$

We would like to investigate the dependence of μ on n . That is, we estimate the magnitude of μ such that (26) holds for all edges. First of all, it is safe to claim that (26) holds for all ij whose $\tilde{s}_{ij}^* > 0.5$ when μ is a positive constant (i.e., $\mu = \Theta(1)$). That is, the left-hand side of (26) is lower bounded by a positive constant. Let $\mathbf{n}_{ij,k}$ be the normal vector of the plane $\text{Span}\{\mathbf{t}_k^* - \mathbf{t}_i^*, \mathbf{t}_k^* - \mathbf{t}_j^*\}$, where \mathbf{t}_i^* follows the standard Gaussian distribution. For $\tilde{s}_{ij}^* \leq 0.5$, one can show that

$$\begin{aligned} \frac{1}{|G_{ij}|} \sum_{k \in G_{ij}} \tilde{d}_{ij,k} &\geq \frac{1}{|G_{ij}|} \sum_{k \in G_{ij}} |\gamma_{ij}^\top \mathbf{n}_{ij,k}| \geq \frac{c'}{\sqrt{\log n}} |\gamma_{ij}^\top \gamma_{ij}^*| \\ &= \frac{c}{\sqrt{\log n}} \min(\tilde{s}_{ij}^*, 1 - \tilde{s}_{ij}^*) = \frac{c}{\sqrt{\log n}} \tilde{s}_{ij}^*, \end{aligned} \quad (27)$$

for some absolute constant c', c , which suggests

$$\mu \geq c/\sqrt{\log n}.$$

In (27), the first inequality follows from the definition of $\tilde{d}_{ij,k}$, the first equality follows from the definition of \tilde{s}_{ij}^* and the last equality is due to the assumption $\tilde{s}_{ij}^* \leq 0.5$. The second inequality is commonly assumed for all $ij \in E$ in [7, 12, 25], which they call the $c/\sqrt{\log n}$ -well-distributed condition. It is proved in [7] that if \mathbf{t}_i^* is i.i.d. with standard Gaussian, then $c/\sqrt{\log n}$ -well-distributed condition holds with high probability.

B Explanation of the Order of Complexity for T-AAB in Table 1

We assume the Erdős–Rényi graph $G(n, p)$, where p is the probability of connecting two nodes, with edge corruption probability q . We show that the recovery guarantee in Theorem 2.1 holds under this probabilistic model, provided

$$p = \Omega(n^{-1/2} \log^{1/2} n) \quad (28)$$

and

$$\epsilon_b = pq = O(p/\sqrt{\log n}). \quad (29)$$

We note that given (28), (29) is equivalent to

$$q = O(1/\sqrt{\log n}). \quad (30)$$

We first verify (30), where we note that it is sufficient to focus on the worst case

$$q = c_1 / \sqrt{\log n} \text{ for an absolute constant } c_1.$$

That is, we show that this choice is sufficient for exact recovery with high probability.

We prove exact recovery by establishing with high probability the sufficient condition of Theorem 2.1

$$\lambda < 1 + \frac{eC_\alpha}{\mu} - \sqrt{\frac{eC_\alpha}{\mu} \left(2 + \frac{eC_\alpha}{\mu}\right)}. \quad (31)$$

We control the ratio of bad cycles as follows. For any fixed edge $ij \in E$, $\lambda_{ij} = |B_{ij}|/|N_{ij}|$ is the average of the Bernoulli random variables $X_k = \mathbf{1}_{\{k \in B_{ij}\}}$ where $k \in B_{ij}$ with probability $1 - (1 - q)^2$. Consequently,

$$\mathbb{E}(\lambda_{ij}) = 1 - (1 - q)^2 = q(2 - q) \leq 2q = \frac{2c_1}{\sqrt{\log n}}.$$

Next, we investigate the concentration bound for λ_{ij} and then for $\lambda = \max_{ij} \lambda_{ij}$. We recall the following one-sided Chernoff bound [4] for independent Bernoulli random variables $\{X_l\}_{l=1}^n$ with means $\{p_l\}_{l=1}^n$, $\bar{p} = \sum_{l=1}^n p_l / n$, and any $\eta \geq 1$:

$$\Pr \left(\frac{1}{n} \sum_{l=1}^n X_l > (1 + \eta) \bar{p} \right) < e^{-\frac{\eta^2}{2+\eta} \bar{p} n}. \quad (32)$$

Applying (32) with the random variables $X_k = \mathbf{1}_{\{k \in B_{ij}\}}$ and $\eta = 1$,

$$\Pr \left(\lambda_{ij} > \frac{4c_1}{\sqrt{\log n}} \right) < e^{-\frac{2c_1}{3} \frac{|N_{ij}|}{\log |N_{ij}|}}. \quad (33)$$

To control the size of $|N_{ij}|$ in above probability bound, we use the following Chernoff bound [4] for i.i.d. Bernoulli random variables $\{X_l\}_{l=1}^m$ with means μ and any $0 < \eta < 1$:

$$\Pr \left(\left| \frac{1}{m} \sum_{l=1}^m X_l - \mu \right| > \eta \mu \right) < 2e^{-\frac{\eta^2}{3} \mu m}. \quad (34)$$

We note that by applying (34) with the random variables $\mathbf{1}_{\{k \in N_{ij}\}}$ and $\eta = 1/2$, we obtain that

$$\Pr \left(N_{ij} < \frac{1}{2} np^2 \right) < 2e^{-\frac{1}{12} np^2}. \quad (35)$$

By combining the bounds in (33) and (35), we have for sufficiently large n

$$\Pr \left(\lambda_{ij} > \frac{4c_1}{\sqrt{\log n}} \right) < e^{-\frac{2c_1}{3} \frac{\frac{1}{2} np^2}{\log \frac{1}{2} np^2}} + 2e^{-\frac{1}{12} np^2} < e^{-\frac{c_1}{4} np^2} + 2e^{-\frac{1}{12} np^2}. \quad (36)$$

By applying a union bound over $ij \in E$, we have

$$\Pr \left(\lambda > \frac{4c_1}{\sqrt{\log n}} \right) < |E| e^{-\frac{c_1}{4} np^2} + 2|E| e^{-\frac{1}{12} np^2}, \quad (37)$$

where $\lambda = \max_{ij} \lambda_{ij}$. Therefore, with $q = c_1 / \sqrt{\log n}$ and sufficiently large n , we have

$$\lambda < \frac{4c_1}{\sqrt{\log n}} \quad \text{w.p.} \quad 1 - |E| e^{-\frac{c_1}{4} np^2} - 2|E| e^{-\frac{1}{12} np^2} \quad (38)$$

Finally, we show for a proper constant c_1 , (31) holds with high probability, and the exact recovery is concluded. We note that the RHS of (31) is lower bounded by

$$\begin{aligned} 1 + \frac{eC_\alpha}{\mu} - \sqrt{\frac{eC_\alpha}{\mu} \left(2 + \frac{eC_\alpha}{\mu}\right)} &= \frac{1}{1 + \frac{eC_\alpha}{\mu} + \sqrt{\frac{eC_\alpha}{\mu} \left(2 + \frac{eC_\alpha}{\mu}\right)}} \\ &\geq \frac{1}{2\sqrt{\frac{eC_\alpha}{\mu} \left(2 + \frac{eC_\alpha}{\mu}\right)}} > \frac{1}{2\left(2 + \frac{eC_\alpha}{\mu}\right)} > \frac{1}{\frac{3eC_\alpha}{\mu}} = \frac{\mu}{3eC_\alpha}. \end{aligned} \quad (39)$$

Combining this estimate of μ with (39), we obtain

$$1 + \frac{eC_\alpha}{\mu} - \sqrt{\frac{eC_\alpha}{\mu} \left(2 + \frac{eC_\alpha}{\mu}\right)} > \frac{c}{3eC_\alpha} \frac{1}{\sqrt{\log n}}. \quad (40)$$

Therefore, to guarantee (31) it suffices to let RHS of (38) be bounded from above by the RHS of (40). Namely, we require that

$$\frac{4c_1}{\sqrt{\log n}} < \frac{c}{3eC_\alpha} \frac{1}{\sqrt{\log n}},$$

which can be easily satisfied by setting $c_1 < \frac{c}{12eC_\alpha}$. Therefore, with the order of $q = (c/(12eC_\alpha\sqrt{\log n}))$ and equivalently $\epsilon_b = (cp/(12eC_\alpha\sqrt{\log n}))$, we can guarantee (31) and hence exact recovery with the probability specified in (38). Consequently, we verify that (30) is sufficient for exact recovery with the latter probability.

We finally note that assuming (28), that is, $p \geq c_0 n^{-1/2} \log^{1/2} n$ for sufficiently large constant c_0 , the probability specified in (38) is high. Indeed,

$$1 - |E|e^{-\frac{c_1}{4}np^2} - 2|E|e^{-\frac{1}{12}np^2} > 1 - 3n^2 \exp(-\min\{\frac{c_1}{4}, \frac{1}{12}\}np^2) \quad (41)$$

$$> 1 - 3n^2 \exp(-\min\{\frac{c_1}{4}, \frac{1}{12}\}c_0 \log n) = 1 - 3n^{2-\min\{\frac{c_1}{4}, \frac{1}{12}\}c_0}. \quad (42)$$

Thus, with high probability, the recovery conditions in Theorem 2.1 are satisfied when (28) and (29) hold. We thus verify the bounds reported for T-AAB in Table I.

C Visualization of 3D sparse models on ETH3D

We compare 3D sparse point cloud reconstructions of Cycle-Sync, GLOMAP and Theia. For GLOMAP and Theia, we use their default reconstruction parameters. For the Cycle-Sync pipeline, we feed the resulting camera poses to the 3D point triangulator in COLMAP (it uses bundle adjustment for the triangulations, while fixing camera pose estimators) and return the sparse 3D model. The latter was done quickly without careful tuning of parameters. Table 2 compares the number of triangulated 3D points for different SfM pipelines and some 3D sparse models on ETH3D. Tables 3 and 4 demonstrate the actual 3D sparse models by these methods.

We observe from Table 2 that for 9 out of the 13 datasets, Cycle-Sync improves the number of triangulated 3D points. This leads to an improvement of the overall quality of reconstruction for these datasets as noticed in Tables 3 and 4. This is due to the improvement on initial camera poses thanks to Cycle-Sync. For the other 4 datasets, Cycle-Sync fails to recover a meaningful 3D sparse model. For two of these datasets (meadow and office) all three methods are not performing well. For one dataset (relief) Theia is the only one which performs well and for the last dataset (relief_2) GLOMAP performs better than the other two methods.

Dataset	Cycle-Sync	GLOMAP	Theia
courtyard	30851	27674	17835
delivery_area	49306	25403	9534
electro	28061	24477	2641
facade	86111	66302	70571
kicker	26649	18685	10232
meadow	647	667	649
office	1351	2686	1395
pipes	8662	3551	1423
playground	16885	11816	416
relief	1642	12617	29588
relief_2	1695	16902	3212
terrace	25285	13898	7216
terrains	47485	25082	12876

Table 2: Number of triangulated 3D points for each dataset using Cycle-Sync, GLOMAP, and Theia

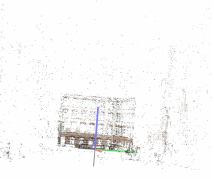

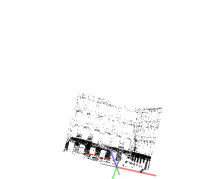
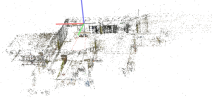
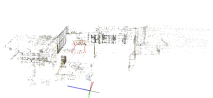

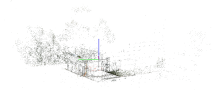
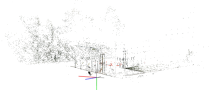
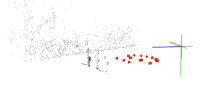
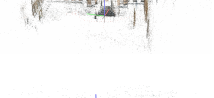

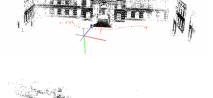
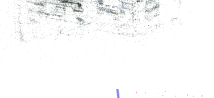
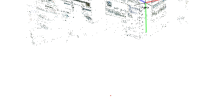



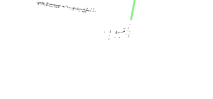
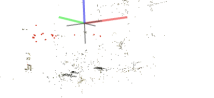

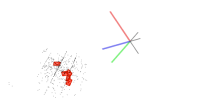


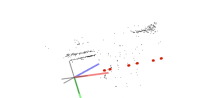


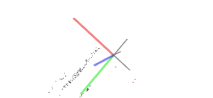
Dataset	Cycle-Sync	GLOMAP	Theia
courtyard			
delivery_area			
electro			
facade			
kicker			
meadow			
office			
pipes			
playground			

Table 3: Triangulated 3D reconstructions (Part 1) using Cycle-Sync, GLOMAP, and Theia.

D Visualization of Camera Pose Estimation

We demonstrate pose estimation results for four scenes: *courtyard*, *meadow*, *office*, and *pipes*. For each scene, we compare the ground-truth camera poses with those estimated by GLOMAP, Theia, and Cycle-Sync. Figures 5-8 illustrate these comparisons. For the first three scenes, Cycle-Sync produces camera poses that align more closely with the ground truth, while GLOMAP and Theia exhibit larger misalignments. In the *pipes* scene, both Cycle-Sync and GLOMAP achieve good alignment, whereas Theia fails to produce meaningful results.

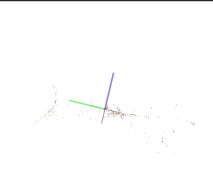
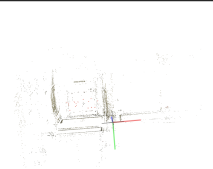
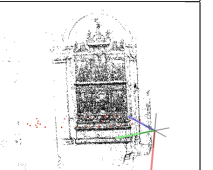
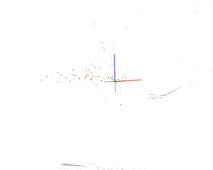
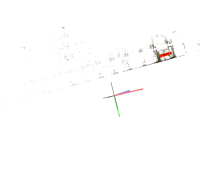
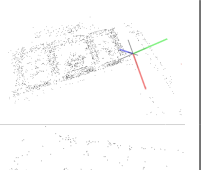
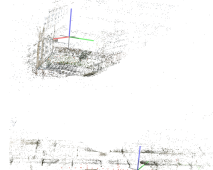

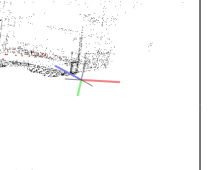
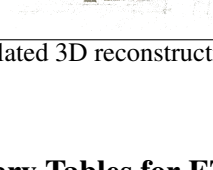
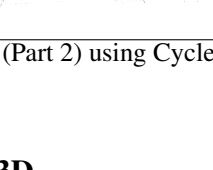
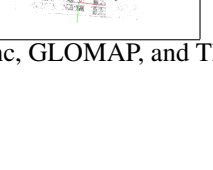
Dataset	Cycle-Sync	GLOMAP	Theia
relief			
relief_2			
terrace			
terrains			

Table 4: Triangulated 3D reconstructions (Part 2) using Cycle-Sync, GLOMAP, and Theia.

E Supplementary Tables for ETH3D

We provide additional tables and figures to demonstrate the pose estimation quality of Cycle-Sync. Table 5 demonstrates the location error for each SfM pipeline. Table 6 demonstrates the location error for different location estimation algorithms. Table 7 demonstrates the effect of each building block of Cycle-Sync by beginning with the LUD pipeline, and gradually adding MPLS, MPLS-cycle, Cycle-Sync and STE.

Table 5: Translation Error of each SfM pipeline on ETH3D. Here \bar{t} and \hat{t} denote the mean translation error and median translation error, respectively. BA refers to bundle adjustment.

Scene	Cycle-Sync		LUD		GLOMAP (with BA)		Theia (with BA)	
	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}
courtyard	0.27	0.02	0.85	0.75	0.34	0.03	0.01	0.01
delivery_area	0.15	0.04	0.37	0.24	0.01	0.00	0.09	0.00
electro	0.21	0.03	0.30	0.10	0.01	0.01	0.01	0.01
facade	0.25	0.00	0.43	0.18	1.01	1.01	0.01	0.00
kicker	0.02	0.01	0.09	0.02	0.36	0.01	0.01	0.01
meadow	0.02	0.02	0.40	0.28	0.91	1.01	0.68	0.51
office	0.20	0.03	0.17	0.03	0.06	0.01	0.95	0.97
pipes	0.01	0.01	0.06	0.03	0.01	0.01	0.01	0.00
playground	0.12	0.01	0.40	0.13	1.30	0.99	0.01	0.00
relief	0.00	0.00	0.00	0.00	0.90	0.78	0.01	0.01
relief_2	0.01	0.01	0.70	0.73	0.01	0.01	0.81	0.79
terrace	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
terrains	0.01	0.01	0.02	0.01	0.00	0.00	0.42	0.05
Average	0.10	0.01	0.29	0.19	0.38	0.30	0.23	0.18

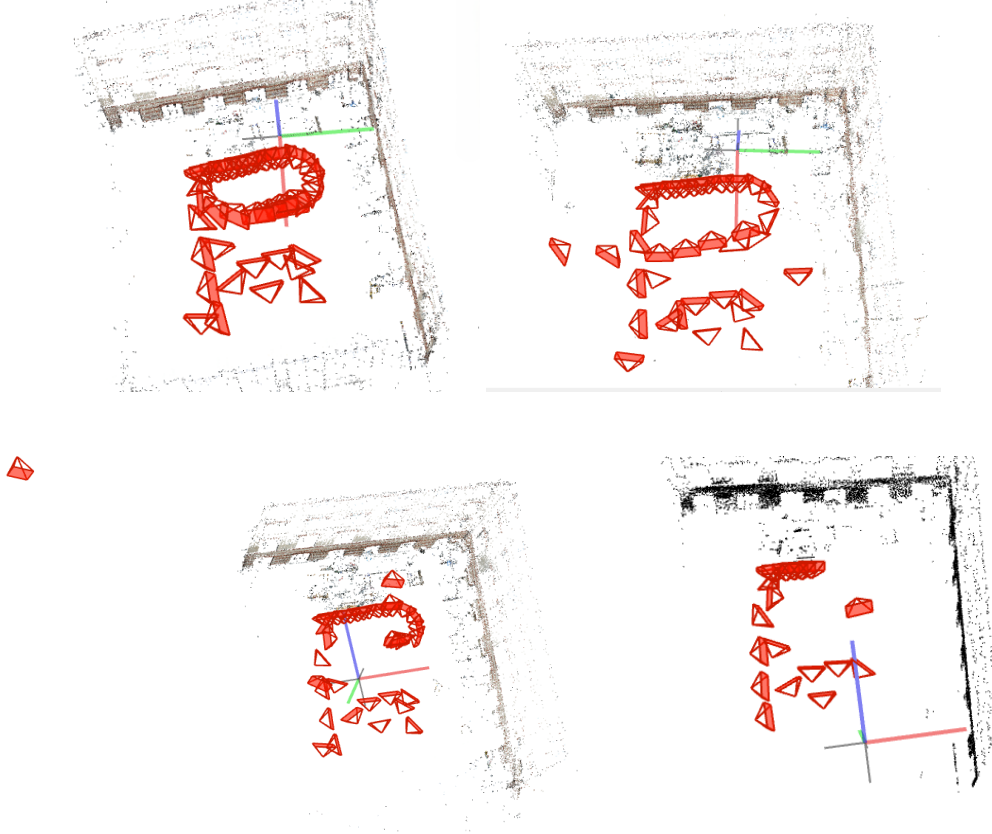


Figure 5: Visualization of camera location estimations and ground truth on the courtyard dataset. Top left: ground truth. Top right: Cycle-Sync. Bottom left: GLOMAP. Bottom right: Theia.

Dataset	LUD		BATA		ShapeFit		FusedTA		Cycle-Sync	
	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}
courtyard	0.37	0.02	0.41	0.08	0.32	0.02	0.21	0.05	0.27	0.02
delivery area	0.25	0.16	0.35	0.23	0.18	0.08	0.96	0.93	0.15	0.04
electro	0.24	0.06	0.23	0.06	0.21	0.02	0.22	0.04	0.21	0.03
facade	0.30	0.01	0.28	0.01	0.98	0.98	0.91	0.86	0.25	0.00
kicker	0.03	0.01	0.03	0.01	0.02	0.01	0.03	0.02	0.02	0.01
meadow	0.06	0.02	0.11	0.05	0.08	0.02	0.08	0.02	0.02	0.02
office	0.20	0.03	0.21	0.03	0.20	0.03	0.21	0.03	0.20	0.03
pipes	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
playground	0.16	0.03	0.15	0.04	0.08	0.01	0.17	0.08	0.12	0.01
relief	0.00	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
relief 2	0.18	0.19	0.04	0.03	0.90	0.90	0.11	0.08	0.01	0.01
terrace	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
terrains	0.01	0.01	0.06	0.06	0.01	0.00	0.02	0.01	0.01	0.01
Average	0.14	0.04	0.15	0.05	0.23	0.16	0.23	0.16	0.10	0.01

Table 6: Comparison of mean (\bar{t}) and median (\hat{t}) translation error for each ETH3D scene for different location estimation algorithms.

F Runtime

Table 8 compares the runtime of location estimation methods on ETH3D. We observe that STE-based methods are significantly faster than non-STE methods, including LUD+IRLS (the old LUD pipeline). In particular, Cycle-Sync runtime is 48% lower than that of the LUD pipeline. Although Cycle-Sync is slower than common location estimation algorithms such as BATA, ShapeFit, and FusedTA, its

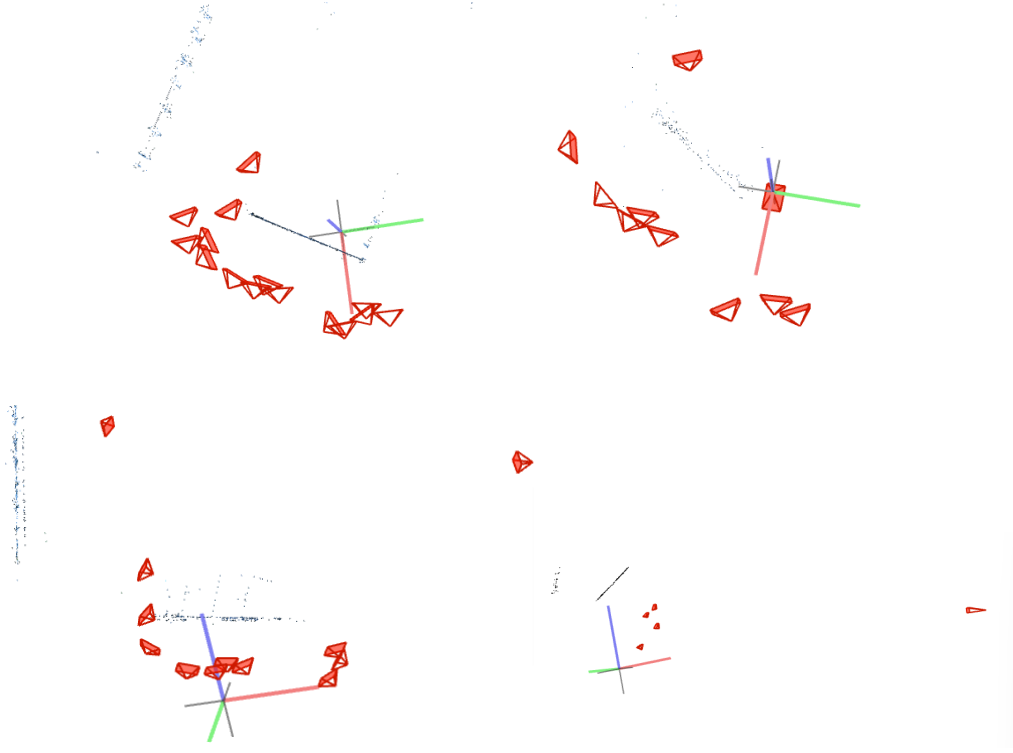


Figure 6: Visualization of camera location estimations and ground truth on the meadow dataset. Top left: ground truth. Top right: Cycle-Sync. Bottom left: GLOMAP. Bottom right: Theia.

Scene	LUD+IRLS		LUD+MPLS		LUD+MPLS-cycle		Cycle-Sync+MPLS-cycle		STE+Cycle-Sync+MPLS-cycle	
	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}
courtyard	0.85	0.75	0.80	0.71	0.82	0.78	0.76	0.39	0.27	0.02
delivery area	0.37	0.24	0.36	0.23	0.37	0.28	0.16	0.06	0.15	0.04
electro	0.30	0.10	0.31	0.11	0.31	0.10	0.24	0.04	0.21	0.03
facade	0.43	0.18	0.49	0.19	0.53	0.04	0.26	0.00	0.25	0.00
kicker	0.09	0.02	0.07	0.03	0.10	0.05	0.03	0.01	0.02	0.01
meadow	0.39	0.28	0.49	0.47	0.48	0.23	0.18	0.06	0.02	0.02
office	0.17	0.03	0.18	0.03	0.22	0.03	0.21	0.03	0.20	0.03
pipes	0.06	0.03	0.05	0.02	0.05	0.02	0.01	0.01	0.01	0.01
playground	0.40	0.13	0.36	0.12	0.42	0.19	0.23	0.01	0.12	0.01
relief	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
relief 2	0.70	0.73	0.15	0.15	0.15	0.15	0.01	0.01	0.01	0.01
terrace	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
terrains	0.02	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.01
Average	0.29	0.19	0.25	0.16	0.27	0.14	0.16	0.05	0.10	0.01

Table 7: Translation errors (\bar{t} = mean translation error, \hat{t} = median translation error) across all methods for ablation study.

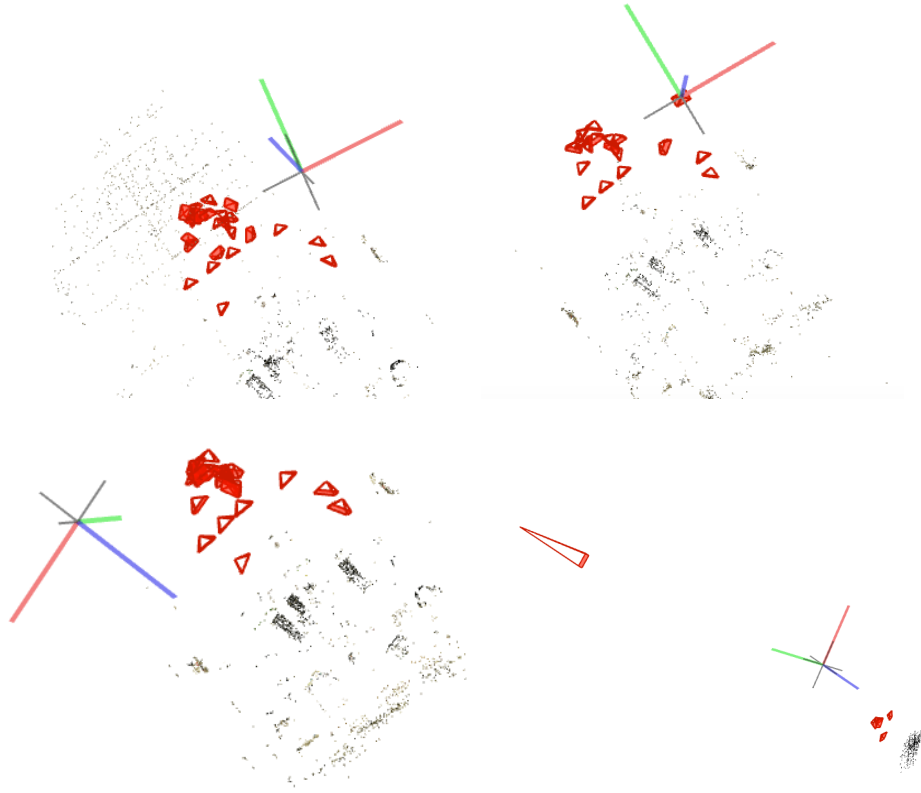


Figure 7: Visualization of camera location estimations and ground truth on the office dataset. Top left: ground truth. Top right: Cycle-Sync. Bottom left: GLOMAP. Bottom right: Theia.

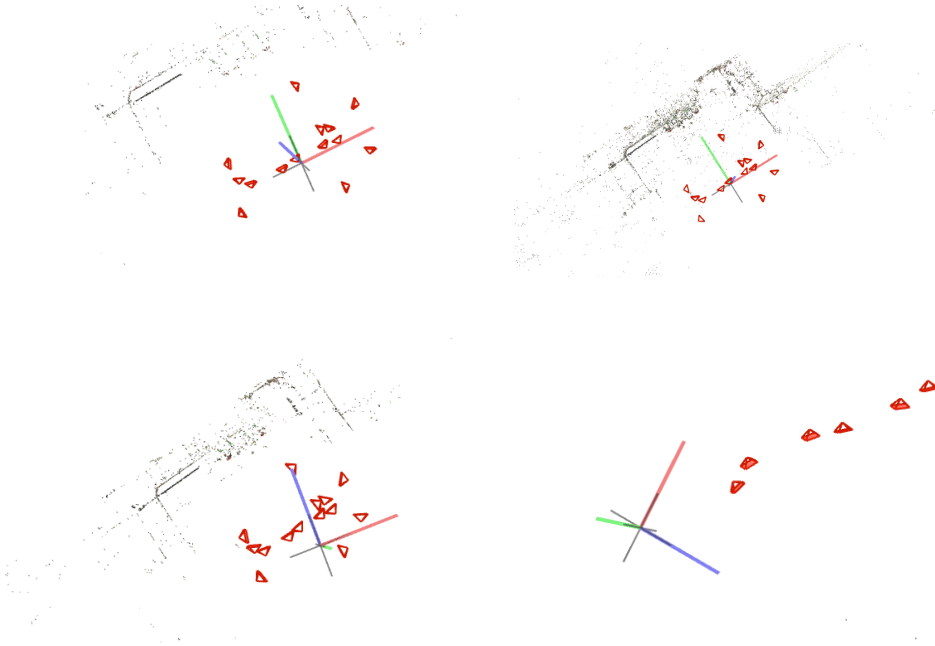


Figure 8: Visualization of camera location estimations and ground truth on the pipes dataset. Top left: ground truth. Top right: Cycle-Sync. Bottom left: GLOMAP. Bottom right: Theia.

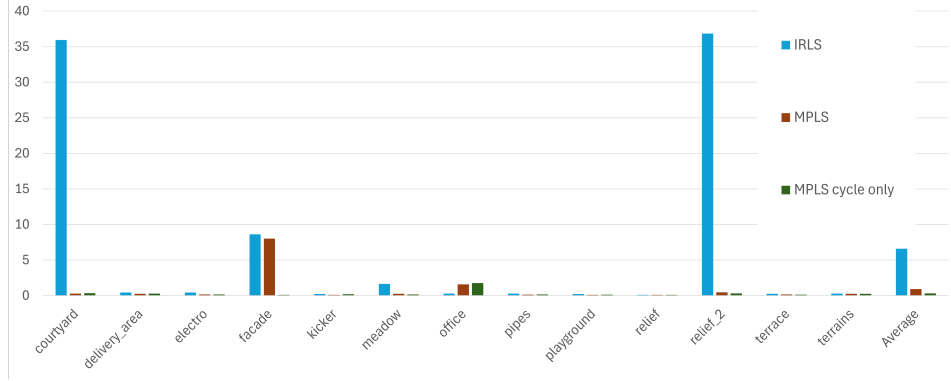


Figure 9: Rotation error (degrees) comparison on ETH3D for different rotation synchronization solvers.

runtime remains within the same order of magnitude, while achieving superior accuracy and stability in camera pose estimation.

Scene	LUD+IRLS	LUD+MPLS	STE-based (with MPLS-cycle)				
			STE+LUD	STE+BATA	STE+ShapeFit	STE+FusedTA	Cycle-Sync
courtyard	23.60	19.38	6.10	5.83	5.74	6.01	10.05
delivery area	16.99	15.97	4.13	4.06	3.95	4.16	8.38
electro	15.58	13.92	4.21	3.92	3.82	4.19	8.58
facade	100.68	89.37	24.48	23.88	24.47	24.32	30.41
kicker	14.31	14.30	3.94	4.90	3.63	3.84	8.32
meadow	0.95	0.92	0.42	0.40	0.41	0.58	4.63
office	1.89	1.70	0.91	0.86	0.87	1.02	5.16
pipes	2.23	2.13	0.73	0.64	0.65	0.77	4.82
playground	10.78	9.60	2.79	2.62	2.57	2.84	7.34
relief	5.49	5.09	1.48	1.54	1.47	1.51	5.68
relief 2	7.93	7.90	2.33	2.19	2.16	2.36	7.05
terrace	8.38	8.53	1.98	1.99	1.89	2.01	6.47
terrains	13.18	12.72	4.37	3.80	3.70	3.96	8.88
Average	17.08	15.50	4.45	4.36	4.26	4.43	8.91

Table 8: Runtime comparison (in seconds) of different SfM pipelines on ETH3D.

G Table and Figures for Rotation Synchronization

In this section we show the tables and figures for rotation errors. Figure 9 demonstrates the rotation errors on ETH3D across different rotation synchronization methods. Table 9 demonstrates the rotation errors on ETH3D across different pipelines.

We observe that MPLS-cycle (used in our Cycle-Sync) greatly improves rotation accuracy over existing pipelines. On average, MPLS-cycle reduces the median rotation error by 62.8% and mean rotation error by 74.1%, compared to the best existing pipeline GLOMAP. Also, our proposed MPLS-cycle reduces the mean rotation error of MPLS by 56.6% and median rotation error by 64.8%. It is worth noting that Cycle-Sync outperforms GLOMAP and Theia even without bundle adjustment. This demonstrates that even without bundle adjustment, our approach outperforms baselines that rely on it.

H Additional Experiment for IMC-PT

In this section we compare the camera location estimation results for different location estimation algorithms on IMC-PT. This dataset consists of 9 city-scale image sets, as well as ground truth camera poses estimated by aligning COLMAP SfM model with a LiDAR scan. We generate image matches using LoFTR [29], a deep learning feature matching method instead of SIFT. We use LoFTR since

Table 9: Comparison of rotation error (degrees) on ETH3D for different pipelines. Here \bar{R} , \hat{R} means the mean rotation error and the median rotation error measured in degrees (0° - 180°) respectively. BA refers to bundle adjustment.

Scene	Cycle-Sync		LUD		GLOMAP (with BA)		Theia (with BA)	
	\bar{R}	\hat{R}	\bar{R}	\hat{R}	\bar{R}	\hat{R}	\bar{R}	\hat{R}
courtyard	0.78	0.36	43.85	35.93	3.50	1.61	0.14	0.10
delivery_area	0.76	0.28	0.78	0.43	0.09	0.07	0.41	0.21
electro	1.11	0.18	1.28	0.45	0.11	0.11	0.07	0.06
facade	0.18	0.11	18.10	8.64	0.78	0.35	0.09	0.09
kicker	0.44	0.21	0.25	0.22	0.47	0.22	0.11	0.09
meadow	0.29	0.18	2.94	1.64	21.13	6.05	3.14	3.37
office	3.39	1.69	8.98	0.29	0.69	0.34	7.23	6.34
pipes	0.17	0.18	0.34	0.28	0.12	0.13	0.08	0.09
playground	0.17	0.14	0.34	0.22	0.67	0.23	0.07	0.09
relief	0.11	0.11	0.11	0.12	3.18	1.68	0.14	0.15
relief_2	0.29	0.31	41.78	36.84	0.10	0.10	36.85	34.23
terrace	0.15	0.13	0.27	0.27	0.12	0.12	0.15	0.13
terrains	0.22	0.25	0.27	0.29	0.19	0.21	3.54	1.78
Average	0.62	0.32	9.18	6.59	2.40	0.86	4.00	3.59

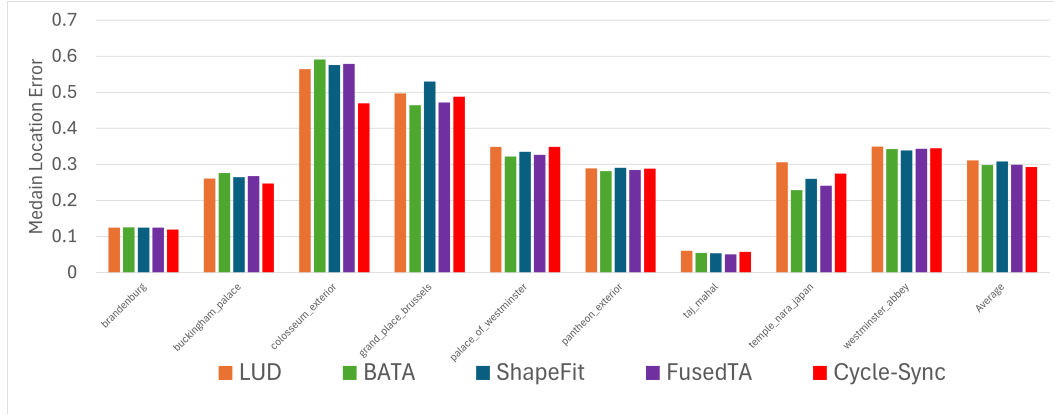


Figure 10: Median translation error for each IMC-PT scene and their average. The last column denotes the average median error across all datasets.

it is proved to be effective on popular homography estimation, relative pose estimation and visual localization benchmarks. Table 10 and Figure 10 demonstrate the location error of different location estimation algorithms, where rotation synchronization method is MPLS-cycle and all methods use STE.

We observe that Cycle-Sync achieves the smallest mean and median location error averaging on all datasets. For 3 out of 9 datasets, Cycle-Sync achieves both the smallest mean and median error. For other datasets, Cycle-Sync achieves no significantly larger mean and median error. The largest difference from Cycle-Sync to the best method in mean and median location error is 0.05, which is small compared to the average scale of location. The improvement of Cycle-Sync is smaller than that in ETH3D. We believe the reason is that LoFTR is not a good feature for this dataset, since it tends to overfit repetitive structures such as windows and facades. To sum up, while the gains over baselines are smaller than on ETH3D, Cycle-Sync still achieves the best mean and median across most scenes.

Table 10: Translation Errors (\bar{t} and \hat{t}) for IMC-PT.

Scene	LUD		BATA		ShapeFit		FusedTA		Cycle-Sync	
	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}	\bar{t}	\hat{t}
brandenburg										
gate	0.24	0.13	0.25	0.13	0.24	0.12	0.24	0.12	0.24	0.12
buckingham										
palace	0.36	0.26	0.37	0.28	0.38	0.26	0.37	0.27	0.35	0.25
colosseum										
exterior	0.75	0.56	0.90	0.59	0.67	0.58	0.92	0.58	0.56	0.47
grand place										
brussels	0.55	0.50	0.54	0.46	0.60	0.53	0.54	0.47	0.54	0.49
palace of										
westminster	0.43	0.35	0.40	0.32	0.44	0.33	0.41	0.33	0.44	0.35
pantheon										
exterior	0.44	0.29	0.44	0.28	0.44	0.29	0.44	0.28	0.44	0.29
taj										
mahal	0.12	0.06	0.12	0.05	0.12	0.05	0.12	0.05	0.12	0.06
temple nara										
japan	0.48	0.31	0.43	0.23	0.45	0.26	0.43	0.24	0.46	0.27
westminster										
abbey	0.87	0.35	0.84	0.34	0.83	0.34	0.84	0.34	0.89	0.35
Average	0.47	0.31	0.48	0.30	0.46	0.31	0.48	0.30	0.45	0.29

I Additional Supplementary Tables for ETH3D

I.1 Sensitivity to Initialization

In tables [12](#) and [11](#) we report the mean and median location error on ETH3D data (averaged over different scenes) after several iterations for T-AAB and trivial initialization schemes. While the T-AAB initialization accelerates convergence by providing better starting weights, it does not significantly influence the final accuracy. Even trivial initialization using uniform weights performs similarly after sufficient iterations. Therefore, our method is robust even to trivial initialization, let alone variations in the T-AAB parameter.

Iteration	Mean Error	Median Error
5	0.110	0.020
10	0.100	0.017
15	0.099	0.016
20	0.099	0.016

Table 11: Performance with trivial (uniform) initialization.

Iteration	Mean Error	Median Error
5	0.102	0.018
10	0.099	0.016
15	0.099	0.015
20	0.099	0.014

Table 12: Performance with T-AAB initialization.

I.2 Replacing LUD with BATA in Our Pipeline

In table [13](#) we show results of replacing LUD with BATA within our reweighting framework. Indeed, integrating LUD under Cycle-Sync’s reweighting leads to better performance compared to integrating BATA. This is likely due to the stronger constraint of LUD for preventing collapsed trivial solution (the constraint is enforced on every edge). Moreover, our Welsh-type objective function already accounts

for large variations in distances, making angle-based methods such as BATA less advantageous in this case.

Method	Mean Error	Median Error
Ours-LUD	0.099	0.014
Ours-BATA	0.202	0.079

Table 13: Comparison between LUD and BATA integration under the Cycle-Sync framework on ETH3D data.

I.3 Annealing Schedule λ_t

In tables 14 and 15 we include synthetic experiments for different annealing schedules λ_t in settings with both additive noise and high corruption. Table 14 uses uniform corruption with $q = 0.7$ and higher noise level $\sigma = 0.2$.

λ_t	$\frac{t}{10+t}$ (ours)	0	$\frac{10}{10+t}$	1	$\frac{t}{t+5}$
median err	0.24	0.36	0.27	0.37	0.29

Table 14: Uniform corruption $q = 0.7$, noise level $\sigma = 0.2$.

Table 15 the adversarial corruption with $q = 0.45$ (close to the theoretical limit $q = 0.5$) and higher noise level $\sigma = 0.2$.

λ_t	$\frac{t}{10+t}$ (ours)	0	$\frac{10}{10+t}$	1	$\frac{t}{t+5}$
median err	0.17	0.32	0.18	0.20	0.17

Table 15: Adversarial corruption $q = 0.45$, noise level $\sigma = 0.2$.

We observe that our choice of λ_t has the lowest median error for both settings. Therefore, our proposed schedule strikes a good balance between residual-driven updates early on and cycle-consistency emphasis later. This schedule has consistently outperformed alternatives, particularly in settings with both additive noise and high corruption.

We remark that using only the residual for reweighting (i.e., setting $\lambda_t = 0$, which corresponds to IRLS) often leads to significantly higher errors compared to cycle-based reweighting methods. The underlying issue is that, in noisy settings, some bad edges may coincidentally exhibit low residuals. When this occurs, the aggressive reweighting imposed by the Welsch objective can assign them disproportionately large weights, thereby amplifying their adverse impact. In contrast, our cycle-based reweighting effectively overcomes this limitation: it is extremely unlikely for a bad edge to exhibit a low average cycle inconsistency, unless all cycles it participates in are consistent—an event that is highly improbable for corrupted measurements. Overall, we find that our annealing strategy consistently achieves the best performance across most scenarios.

We also observe this trend in the context of rotation synchronization. Figure G illustrates that emphasizing cycle-consistency can significantly reduce orientation error. However, for rotation there is no need for annealing as it does not rely on distance estimation.

J Additional Experiment on 1DSfM Datasets

We report the mean and median location errors for 1DSfM dataset (averaged over different scenes), with all methods consistently preprocessed using STE and MPLS-cycle in table 16. Note that the ground truth camera poses in 1DSfM are generated by Bundler, which is considered outdated compared to modern tools like COLMAP (and even COLMAP may lack the accuracy of laser scans). We remark that this could be less reliable when benchmarking high-precision location solvers.

Method	Mean Error	Median Error
Ours	0.292	0.115
LUD	0.314	0.132
BATA	0.362	0.130
ShapeFit	0.670	0.410
FusedTA	0.330	0.130

Table 16: Performance comparison on the 1DSfM (photo tourism) datasets, averaged over multiple scenes. All methods are processed with STE and MPLS-cycle.

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