
Matching Markets Meet LLMs: Algorithmic Reasoning with Ranked Preferences

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Abstract

1 The rise of Large Language Models (LLMs) has driven progress in reasoning tasks—
2 from program synthesis to scientific hypothesis generation—yet their ability to
3 handle ranked preferences and structured algorithms in combinatorial domains
4 remains underexplored. We study matching markets, a core framework behind
5 applications like resource allocation and ride-sharing, which require reconciling
6 individual ranked preferences to ensure stable outcomes. We evaluate seven state-
7 of-the-art models on a hierarchy of preference-based reasoning tasks—ranging from
8 stable-matching generation to instability detection, instability resolution, and fine-
9 grained preference queries—to systematically expose their logical and algorithmic
10 limitations in handling ranked inputs. Surprisingly, even top-performing models
11 with advanced reasoning struggle to resolve instability in large markets, often
12 failing to identify blocking pairs or execute algorithms iteratively. We further show
13 that *parameter-efficient fine-tuning* (LoRA) significantly improves performance in
14 small markets, but fails to bring about a similar improvement on large instances,
15 suggesting the need for more sophisticated strategies to improve LLMs’ reasoning
16 with larger-context inputs.

17 1 Introduction

18 The emergence of Large Language Models (LLMs) has positioned them as integral components in a
19 wide range of reasoning-intensive tasks such as program synthesis, logical inference, mathematical
20 problem solving, and scientific hypothesis generation, highlighting the importance of structured
21 problem-solving capabilities. Despite their recent success in symbolic and logical reasoning, their
22 capacity to reason over ranked preferences and to execute structured algorithms within combinatorial
23 domains remains largely unexplored. Preference reasoning constitutes a foundational component
24 in numerous domains, including economic contexts—e.g., auctions, voting systems, and market
25 design—and in the architecture of pre-trained generative models using Reinforcement Learning from
26 Human Feedback (RLHF) to capture and internalize human value judgments. These methods often
27 have to execute algorithms on a large number of preference lists (either pairwise, partial, or complete
28 rankings) to aggregate the rankings through constitutional AI [2] or social choice theory [15].

29 Despite substantial progress, reasoning over preferences remains a non-trivial endeavor: ensuring
30 transitivity [60, 71], accurately augmenting ordinal rankings [23], and achieving coherent value
31 alignment pose significant challenges. Without robust mechanisms for preference elicitation and the
32 capacity to execute the requisite combinatorial procedures, even state-of-the-art LLMs may produce
33 outputs that diverge from true human preferences [32] or fail to satisfy desirable properties [24].

34 We consider matching markets, a domain that constitutes a fundamental class of problems underlying
35 diverse applications—from healthcare resource allocation to ride-sharing platforms and recommender
36 systems—and demand accurate comprehension of individual preferences and reconciliation of

37 conflicting choices to guarantee system-wide stability. Matching markets are a compelling domain
 38 for studying reasoning in AI models for two key reasons: First, they provide a structured platform
 39 for evaluating reasoning over ranked preferences and algorithmic thinking. Second, LLMs are
 40 increasingly utilized as black-box systems in a variety of economic, social, or medical settings to
 41 inform automated screening in recruitment pipelines [27], investigating market behavior [36], market
 42 clearing in ride-hailing platforms [39], and in general simulating economic interactions [31].

43 1.1 Our Results

44 We focus on four preference-based tasks: (i) *generating* stable solutions, requiring LLMs to produce
 45 stable matchings directly from ranked inputs, (ii) *instability resolution*, demanding preference
 46 reasoning to transform unstable matchings to stable ones, (iii) *instability detection*, in which models
 47 detect blocking pairs within a proposed solution, and (iv) *preference reasoning*, assessing nuanced
 48 query answering over ranked lists. We evaluate seven large language models with varying reasoning
 49 capabilities, including basic models, those with some limited reasoning, and advanced reasoning
 50 models. The following points highlight our contributions and core findings:

51 **Benchmark.** We introduce a benchmark with instances and questions aimed at evaluating the above
 52 tasks involving reasoning over ranked preferences. These tasks are categorized into three levels of
 53 difficulty—Easy, Medium, and Hard—based on problem size. Each task utilizes ranked preferences
 54 sampled from two statistical distribution models: Impartial Culture (IC) and Master List (ML).

55 **Generating Stable Solutions.** Although models with advanced-reasoning capabilities generally
 56 outperform other LLMs on Easy and Medium instances, *all* models struggle to generate stable
 57 solutions on Hard instances—indicating that the combinatorial reasoning capability of LLMs does not
 58 necessarily extend to larger-context inputs. Interestingly, the fraction of invalid and failed solutions
 59 is significantly lower for models with higher reasoning abilities, indicating their understanding of
 60 constraints, despite their inability to perform precise and step-by-step reasoning with preferences.

61 **Instability Detections and Resolution.** We find that LLMs frequently make mistakes in determining
 62 whether solutions are stable, with hallucinations about blocking pairs being the most common among
 63 basic models. Additionally, LLMs’ ability to *correct* unstable solutions is (at best) as good as their
 64 ability to generate them from scratch, in some cases making the provided incorrect solutions worse.

65 **Preference Reasoning.** We consider tasks based on three levels of inference over ranked preferences.
 66 Large language models with advanced reasoning capabilities generally demonstrate a strong com-
 67 prehension of preferences across levels of inference. However, even small errors compound in tasks
 68 requiring multi-step sequences of reasoning (e.g., generating stable solutions or resolving instability),
 69 or in other words, small errors multiply!

70 **Supervised Parameter-Efficient Fine-Tuning.** We demonstrate that fine-tuning an open-source
 71 reasoning model using synthetically generated reasoning traces substantially improves performance,
 72 significantly outperforming advanced-reasoning models on Easy and Medium instances. However,
 73 we find that this approach does not address the challenges LLMs face with large inputs (Hard).

74 1.2 Related Work

75 **Reasoning Capabilities of LLMs.** Mathematical problem solving has been a key area of focus in
 76 evaluating the reasoning ability of LLMs, through a variety of benchmarks such as [14, 29, 30, 54].
 77 LLMs have also demonstrated remarkable capabilities on coding benchmarks such as SWE-Bench
 78 [38] and CodeForces. As SOTA benchmark scores improve, recent work studies whether these
 79 improvements reflect genuine logical reasoning through benchmarks assessing logical consistency
 80 [47] and rule understanding/execution/planning [28]. Furthermore, the recent rise of LLM agents
 81 has increased interest in benchmarking LLMs’ causal reasoning [12] and strategic planning abilities
 82 [20, 37, 62]. Additionally, the emergence of reasoning models has led to benchmarks evaluating
 83 these models’ improved reasoning and planning abilities [10, 46].

84 **Enhancing Reasoning Capabilities of LLMs.** Specialized prompting strategies like Chain-of-
 85 Thought (CoT) [68], Tree-of-Thought (ToT) [73], and Graph-of-Thought (GoT) [3] have performance
 86 abilities on a variety of reasoning benchmarks. Fine-tuning has also been demonstrated to improve
 87 CoT in model outputs [74], as well as economic rationality [11] and abstract reasoning [70]. Addi-
 88 tionally, instruction-tuning has been shown to enhance reasoning in several works [8, 45, 48, 67].

More advanced techniques build upon CoT [64, 76], or utilize multi-agent architectures that leverage cooperative LLMs [65, 75]. More recently, reinforcement-learning (GRPO) has been used to improve model reasoning, the most popular example being the Deepseek-R1 reasoning model [17].

LLMs in Social and Economic Decision Making. While still an emerging area of research, multiple works have focused on the collective decision-making capabilities of LLMs. One particular area of interest is the use of LLMs in preference elicitation [34, 61]. Fish et al. [24] benchmark the ability of models to learn and strategize in unknown economic environments using deliberate exploration. Another notable avenue of work is the study of how well LLMs can represent humans in collective decision-making, an understudied component of LLM alignment [32, 72].

2 Methodology

2.1 Problem Formulation

A *two-sided matching market* consists of two disjoint sets of agents (e.g., riders and drivers, freelancers and job requesters, and content creators and ads) denoted by M and W , where $|M| = |W| = n$. The preference list of an agent i , denoted by \succ_i , is a ranked order list over the agents on the other side. A *preference profile*, \succ , denotes the collection of preferences of all agents. Whenever it is necessary, we write $w_1 \succeq_m w_2$ to denote “either $w_1 \succ_m w_2$ or $w_1 = w_2$ ”. In this paper, we primarily consider the standard model, which assumes a complete and strict preference list (no ties) and aims at finding a *one-to-one* matching between the agents in two sets.¹

Matching and Stability. A *matching* is a function $\mu : M \cup W \rightarrow M \cup W$ such that $\mu(m) \in W$ for all $m \in M$, $\mu(w) \in M$ for all $w \in W$, and $\mu(m) = w$ if and only if $\mu(w) = m$. Given a matching μ , a *blocking pair* with respect to the preference profile \succ is a pair (m, w) who prefer each other over their assigned partners in μ , i.e., $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$. A matching is said to be *stable* if it does not have any blocking pairs. Given an instance of the problem, the set of all possible stable solutions forms a distributive lattice and can be *exponential* in size [42].

In their seminal work, Gale and Shapley [26] proposed an iterative procedure—the *deferred acceptance* algorithm (DA)—that always guarantees to find a stable solution. It proceeds by a series of proposals and rejections. In the initial *proposal* phase, each of the unmatched agents on one side (aka *proposers*) proposes to their favorite agent from the other side (aka *receivers*) according to their preference list. In the subsequent *rejection phase*, each agent on the receiving side tentatively accepts its preferred proposal, rejecting the others. The algorithm terminates when no further proposals can be made. The details of this (as well as additional) algorithms can be found in Appendix C. This underlined solution is simultaneously *optimal* for the proposing side and *pessimal* for the receiving side [53]. We refer to the former as the **Optimal** matching and the latter as the **Pessimal** matching.

Example 1 (An instance with multiple stable solutions.). A *preference profile for a sample instance of size $n = 4$* ; *underlined agents indicate the Optimal matching, the Pessimal matching is indicated with a $*$, and the \dagger indicates a stable matching that is different from the first two.*

$$\begin{array}{llll}
 m_1 : \underline{w_4} & w_3 & w_1^{*,\dagger} & w_2 \\
 m_2 : \underline{w_3} & w_4^\dagger & w_2^* & w_1 \\
 m_3 : \underline{w_1} & w_3^\dagger & w_2 & w_4^* \\
 m_4 : w_1 & \underline{w_2^\dagger} & w_3^* & w_4
 \end{array}
 \quad
 \begin{array}{llll}
 w_1 : m_2 & m_1^{*,\dagger} & \underline{m_3} & m_4 \\
 w_2 : m_2^* & m_3 & \underline{m_4^\dagger} & m_1 \\
 w_3 : m_4^* & m_3^\dagger & m_1 & \underline{m_2} \\
 w_4 : m_4 & m_3^* & m_2^\dagger & \underline{m_1}
 \end{array}$$

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2.2 Dataset, Models, and Setup

Preference Instances. We synthetically sample a set of 300 preference profiles, partitioned into three 100 instances for each *difficulty level*, namely **Easy** ($n = 10$ agents on each side of the market), **Medium** ($n = 20$), and **Hard** ($n = 50$). The preference profiles are sampled from two types of distributions **Impartial Culture** (IC) and **Master-list** (ML), each constituting 50 questions at each difficulty level. An impartial culture (IC) is a well-studied probabilistic model for generating preference profiles in which every agent’s strict preference ranking is drawn independently and

¹This is the standard model considered by the seminal works of Gale and Shapley [26] and Knuth [42].

uniformly at random [4, 22]. It has been extensively studied in the context of economics, matching, and voting theory [5, 7, 9, 66]. A profile with a master-list (ML) is a highly structured preference profile in which all agents on one side of the market share exactly the same strict ranking over the agents on the other side. They represent the *homogeneity* in settings ranging from the labor market to organ allocation in healthcare [6, 35, 40]. While an arbitrary instance generated by IC may admit *exponentially* many stable solutions [42], with a master-list, only a single *unique* stable solution exists, indicating a difficulty level proportional to the size of the space of stable outcomes.

Matching Dataset. We curate a dataset comprising 2850 questions derived from the instances described above.² These questions cover four *task categories*, each applied to the same pool of profiles to ensure consistency: T1: **generating stable solutions**, given a preference profile (300 questions), T2: **instability detection**, given a profile and a solution (1050 questions), T3: **instability resolution**, given a profile and an unstable matching, and T4: **preference reasoning**, given a single preference list or a profile (900 questions).

Models. We select a representative suite of both open-source and closed-source models for evaluation. Since our benchmark is based on a reasoning task, we categorize models by their reasoning ability. We evaluate two **basic** models (those not specifically trained for reasoning), namely Llama-3.3-70B [21] and Gemini-2.0-Flash [57], and five **reasoning** models, namely Qwen-QwQ-32B [63], DeepSeek-70B (Llama-distilled) [17], OpenAI o3-mini [56], DeepSeek-R1 [17], and Gemini-2.5-Pro [16]. Among reasoning models, we classify the last three as **advanced reasoning** models, based on their SOTA performance on reasoning benchmarks [46].

Prompting. The prompt for each task consists of the preference profile for a given instance, followed by task-specific instructions (e.g., computing the “proposer-optimal” matching, or resolving a given unstable matching). To scale up the verification of solution correctness, we instruct LLMs to adhere to a predefined format to report answers. Additionally, we allow LLMs *two re-tries* to correct solutions that are either invalid, partial, or do not adhere to the specified format. See Appendix H for details about the inference setup, and Appendix I for sample prompts.

2.3 Evaluation Criteria

We consider several metrics for evaluating the quality of returned responses depending on the task. To account for cases in which LLM outputs violate task requirements, we categorize responses into the following types: A solution is **invalid** if *some* agent from one side is matched to more than one agent from the other side. It is **partial** if it is not invalid, but some agents remain unmatched. A matching is **stable** if it is a *perfect* one-to-one matching that admits no blocking pair. Otherwise, it is **unstable** if it matches all the agents but admits a blocking pair. The following metrics apply primarily to valid responses. Informally, these metrics measure the distance from a reference stable outcome.

Instability Rate (IR): The instability rate measures the proportion of agents involved in blocking pairs, and thus the degree to which a matching violates the stability criterion. Given a complete matching, instability rate measures the percentage of unstable agents, i.e., those involved in at least one blocking pair. Formally, $IR(\mu, \succ) = \frac{|\{i \in M \cup W \text{ s.t. } j \succ_i \mu(i) \wedge i \succ_j \mu(j) \text{ for some } j \in M \cup W\}|}{2n}$.

Optimality/Pessimality Rate: This rate assesses the overlap between the model’s matching and a reference stable matching, thereby capturing how closely the model’s output mirrors the stepwise proposals and acceptances of a canonical algorithm. Formally, given two perfect matchings, μ and μ' , in a one-to-one market where each matching is viewed as a set of unordered pairs between agents, the Jaccard similarity is defined as $JS(\mu, \mu') = \frac{|\mu \cap \mu'|}{|\mu \cup \mu'|}$. Then, we define **optimality rate** (OR) of a stable matching as its similarity to the proposer-optimal stable solution, which is unique. Similarly, we measure similarity to the Pessimal stable matching. When an instance has a unique stable solution (e.g., profiles with a master-list), optimality and pessimality rates coincide.

3 Generating Stable Solutions

The first task involves evaluating LLMs’ abilities to generate valid, stable matchings in markets with various difficulty levels. This task ideally requires models to reason over ranked preferences while iteratively executing a structured algorithm.

²Our novel dataset, along with code and preference instances, will be made publicly available.

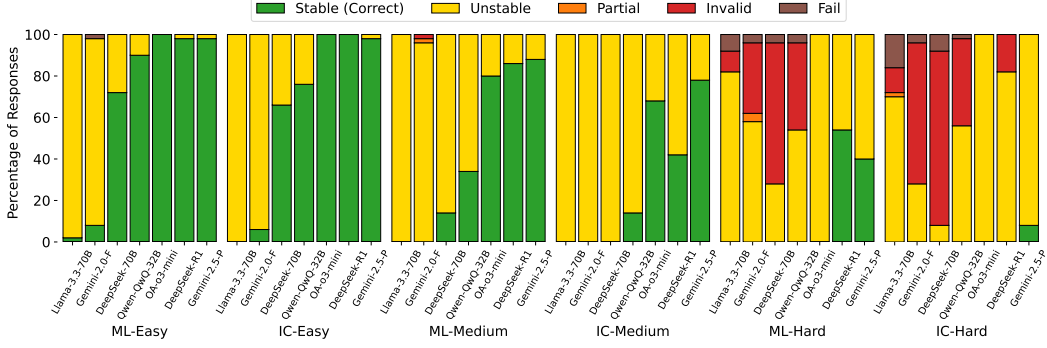


Figure 1: The generated responses by LLMs with Master-List (ML) and Impartial Culture (IC) preferences at different difficulty levels. *Stable* indicates one-to-one matchings with no blocking pairs; otherwise it is *unstable*. *Invalid* do not adhere to one-to-one constraint, *partial* are one-to-one but leave some unmatched, and *Fail* indicates models’ failure to return any matching.

We considered two sub-categories for generating matchings depending on declarative knowledge about algorithms: i) prompt without specifying any algorithm, and ii) prompt with exact step-by-step instructions of how to execute the DA algorithm [26] (see Section 2.1 for details). Our ablation studies showed that the above prompting strategies did not result in qualitatively different outcomes, as all models were able to correctly identify the requirement for considering preferences, the DA algorithm, and its execution steps.³ The detailed results are presented in Appendix D.

Difficulty, Model Size, and Reasoning. Figure 1 demonstrates the performance of the models in generating stable outcomes. Baseline models without explicit reasoning mechanisms are unable to solve even Easy instances, whereas reasoning-enabled models achieve high accuracy on Easy instances but suffer dramatic performance drops on Hard instances. Furthermore, for Hard problems, even advanced reasoning models frequently produce invalid outputs or fail to return any solution. Interestingly, Qwen-QwQ-32B significantly outperforms a much larger model, DeepSeek-70B, indicating that LLMs’ combinatorial reasoning capability does not necessarily scale with model size.⁴

IC vs. ML Profiles. Under Impartial Culture (IC) profiles, the number of stable solutions can grow exponentially as the problem scales (increase of n) [42]. This combinatorial explosion poses a significant challenge for LLMs attempting to identify stable solutions, especially when solely using implicit reasoning over preference lists (without executing a concrete matching algorithm). In contrast, master-list profiles (ML)—irrespective of the underlying sampling method used to generate preferences—admit exactly *one* stable solution. Moreover, this unique stable matching can be constructed in $\mathcal{O}(n)$ steps by (i) extracting the common master list and then (ii) pairing agents in order of their shared priority [35]. See the details of the algorithm in Appendix C.

We observe that there is a significant performance gap between ML and IC instances—this disparity is especially marked for DeepSeek-R1. With IC profiles and Hard instances, *all* models are unable to compute a stable solution.⁵ They perform slightly better under ML profiles, and while this performance drops for Hard instances, these models almost never return invalid or partial matchings.

Prompting Techniques. Prompt-engineering techniques have been empirically demonstrated to enhance the performance of LLMs on mathematical reasoning and formal logic inference tasks [3, 68, 73]. We evaluated a range of prompt-engineering techniques—including few-shot prompting and Chain-of-Thought (CoT) prompting, which supply exemplar “thought processes” and intermediate reasoning steps—in an attempt to bolster LLM performance. However, none of these strategies qualitatively improved on medium- or hard-difficulty instances. See Appendix D for further details.

³Furthermore, converting the *stable-marriage* setting to a *task-scheduling* setting [24], where “men” and “women” are replaced by “workers” and “tasks” (respectively), does not have a significant impact on performance.

⁴Throughout the paper, all statistical comparisons between the percentages of stable solutions returned (across LLMs or across treatments) are done using Fisher’s Exact test [25]. Similarly, any two distributions of Instability or Optimality Rate are statistically compared using Welch’s T-test [69].

⁵Gemini-2.5-Pro is the only model with a positive success rate (= 8%) with IC preferences Hard instances.

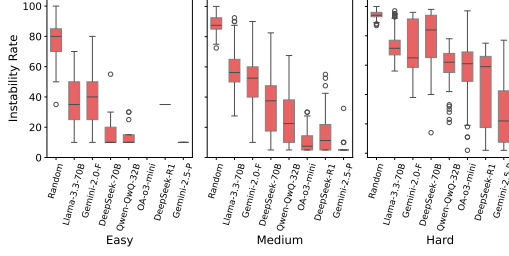


Figure 2: Instability Rate (lower is better) within unstable outcomes returned by each model as compared to randomly selected valid (but not necessarily stable) solutions (Random).

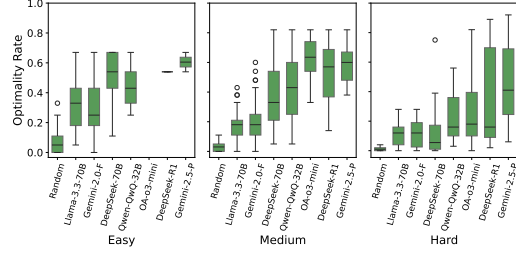


Figure 3: Optimality Rate within unstable outcomes returned by each model as compared to randomly selected valid (but not necessarily stable) solutions (Random).

3.1 Measuring Instability

A natural question is how far LLM-produced responses deviate from stable outcomes. To quantify this, we use two complementary metrics: instability rate and optimality rates (see Section 2.2). The **instability rate** directly reflects the distance from any stable solution, whereas the **optimality rate** implicitly evaluates the model’s success in executing the underlying matching procedure. Figure 2 and Figure 3 illustrate comparisons of LLMs with a baseline of randomly generated outcomes.⁶

Broadly, the advanced-reasoning models generate significantly closer approximations to stability and optimality than their non-reasoning counterparts. Moreover, all evaluated LLMs (regardless of reasoning sophistication) substantially outperform random baselines on both metrics, indicating that they inherently leverage preference structures and exhibit nontrivial reasoning about ranked inputs.

Interestingly, the performance distinction between basic and reasoning models becomes less clear. While the intermediate reasoning models return a lower instability rate in Easy and Medium problems, their performance significantly drops in larger-scale problems (Hard). In fact, the performance of DeepSeek-70B becomes worse than even basic non-reasoning models. We attribute this behavior to the model’s diminished capacity for handling larger input lengths, a hypothesis supported by their lower proportion of valid outcomes (as seen in Figure 1).

4 Resolving Instability

Generating stable solutions requires both exact reasoning over agents’ preference lists and the execution of a stability-guaranteeing procedure (e.g. the DA algorithm). As demonstrated in Section 3, all evaluated models—irrespective of their reasoning capabilities—exhibit severe performance degradation as the problem size grows. This leads to the natural question of whether these models can resolve instability in a given matching—a task that entails detecting blocking pairs through preference reasoning and applying an appropriate sequence of adjustments to restore stability.

We provide LLMs with unstable (but valid) matchings along with preference profiles, and instruct them to convert these initialized solutions to stable matchings. To assess how the instability rate may influence solution quality, we distinguish two classes of initial matchings: **One-BP**, matchings containing exactly one blocking pair (“almost stable”) such that their stability may be resolved through simpler proposal-rejection iterations. See Appendix E for detailed steps and pseudo-code for generating one-BP and random initialization. **Random**, matchings sampled uniformly at random from the set of all valid one-to-one pairings, which typically contain a large number of blocking pairs and thus exhibit high degrees of instability. Note that starting from an arbitrary matching, sequentially resolving blocking pairs may result in a cycle—as shown by Knuth [41]. However, a random sequence converges to stability with probability one [1, 58].

Table 1 displays the fraction of responses in which LLM returns stable matchings when asked to resolve the above types of unstable matching. Surprisingly, our experiments illustrate that in the task of resolving instability, the performance of all evaluated models does not exceed—and even degrades—their performance in generating stable solutions. This behavior persists regardless of initial matchings

⁶Note that the plots only illustrate unstable but valid one-to-one outputs.

Table 1: The percentage of stable matchings returned when tasked with resolving instability starting from One-BP or Random matchings. The numbers in bold represent the highest accuracy (across all LLMs) of resolving the corresponding type of unstable matching.

Difficulty	Preference	Basic LLMs				Reasoning LLMs				Advanced Reasoning LLMs					
		Gemini-2.0-Flash		Llama-3.3-70B		Qwen-QwQ-32B		DeepSeek-70B		o3-mini		DeepSeek-R1		Gemini-2.5-Pro	
		One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random
Easy	IC	2	2	2	0	60	36	46	54	100	100	96	98	96	92
	ML	4	2	0	0	88	78	68	62	96	100	100	98	100	98
Medium	IC	0	0	0	0	22	0	10	0	64	64	28	32	74	60
	ML	0	0	0	0	17	7	20	6	82	78	88	76	80	82
Hard	IC	0	0	0	0	4	0	0	0	0	0	0	0	2	2
	ML	0	0	0	0	0	0	0	0	6	0	28	24	16	34
Average		1.00	0.67	0.33	0.00	31.83	20.16	24.00	20.33	58.00	57.00	56.67	54.67	61.33	61.33

(One-BP or Random) and LLMs’ reasoning capability. In fact, on Hard instances, the output returned by advanced reasoning models on One-BP matchings (i.e., containing a single blocking pair) contains substantially more than one blocking pair. In other words, even for the most basic instances, LLMs often introduce additional instabilities beyond the original single violation. We elaborate on this in Appendix E, demonstrating how all models, including those with advanced reasoning, often return solutions with a higher instability rate, highlighting their inability to systematically eliminate blocking pairs in accordance with preference lists.

5 Detecting Instability

The findings in previous sections raise the question of whether LLMs can reliably detect instability in a given matching—a simpler task that involves iterating over each unmatched pair to determine whether both agents prefer one another over their assigned partners. This procedure requires only a straightforward comparison of preferences and requires $\mathcal{O}(n^2)$ steps.

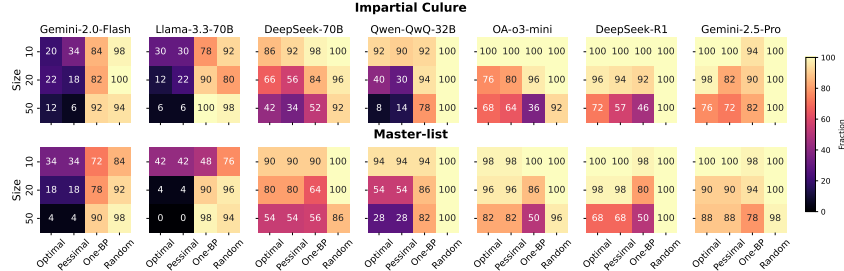


Figure 4: The fraction of responses where each model correctly detects stability or instability of a given matching.

For this task, we evaluate the performance of valid one-to-one matchings initialized under two instability conditions: (i) One-BP, representing nearly stable matchings containing a single blocking pair, and (ii) Random, representing highly unstable matchings with numerous blocking pairs. To detect false-positives, we additionally include two extreme cases of stable matchings: the proposer-optimal (Optimal) and the proposer-pessimal (Pessimal) stable solutions.

Reasoning Models: Preferences and Blocking Pairs. Figure 4 reveals an interesting observation about reasoning models: their performance is influenced by the number of blocking pairs in the matching being evaluated—similar to the observations in Section 4. They achieve a high accuracy on identifying random matchings (which have a larger number of blocking pairs) as unstable and a significantly lower accuracy with matchings that have at most one blocking pair.

Basic Models and Hallucination. Interestingly, the non-reasoning models achieve a high accuracy (80%) with both types of unstable matchings, and extremely low (20%) accuracy with stable matchings. Note that the performance across all models is qualitatively similar in ML and IC profiles, even though each ML profile admits a unique stable solution (thus, identical reports for Optimal and Pessimal). See Appendix F for further analysis. A manual analysis of non-reasoning models uncovers frequent hallucinations about blocking pairs, resulting in a systematic bias toward classifying matchings as unstable. This can be largely attributed to misinterpretations of the input preferences.

6 Reasoning about Ranked Preferences

Many advanced reasoning paradigms, ranging from causal inference [12] and counterfactual analysis to game-theoretic decision making, depend fundamentally on the ability to compare and evaluate alternative choices. As demonstrated thus far, even advanced reasoning models often fail to execute the step-by-step procedures of combinatorial algorithms when those procedures operate over ranked preference lists. This shortcoming motivates the question of whether current LLMs can truly reason *about* preferences, as opposed to merely applying preferences in generating responses heuristically.

To investigate preference comprehension, we introduce a suite of tasks spanning three levels of inference over ranked preferences: (i) **basic retrieval (L1)**, in which models must extract individual preference relations; (ii) **comparison queries (L2)**, requiring pairwise preference judgments; and (iii) **proposal-acceptance simulations (L3)**, which combine comparison of alternatives with binary accept/reject decisions mirroring the dynamics of deferred-acceptance algorithms.

Hierarchical, level-wise reasoning evaluations have been proposed recently in domains such as causal inference of LLMs [12]. For example, an **L1** question is “Who is agent W5’s, 4th-most preferred agent?”, and an **L2** question ‘Would agent W5, prefer M8 over M7?’”



Figure 5: Accuracy on questions about provided preferences, with both IC and ML instances.

Basic models have low accuracy in all levels of difficulty even on small instances, which is probably the reason behind their inability to compute stable solutions (as discussed in Section 5). In basic models (e.g. Llama-3.3-70B) and medium-sized open-source models with reasoning (e.g. DeepSeek-70B), the size of the problem has a greater impact on accuracy as compared to the level of the question, indicating their difficulty of handling larger inputs. Although advanced-reasoning models have significantly higher accuracy rates compared to other models, they still make minor errors, especially in larger profiles (Hard). These instances often require a larger number of reasoning steps over many preference lists, causing minor errors to compound (small errors multiply!).

7 Performance Improvement through LoRA Fine-Tuning

Supervised fine-tuning has proven to be effective in enhancing the reasoning capabilities of LLMs in a variety of tasks, including mathematical problem solving [13, 44], logical reasoning [55], and code generation [49]. Markeeva et al. [52] demonstrate how fine-tuning a small LLM (2B parameters) can significantly improve LLMs’ ability to execute textbook algorithms (e.g., sorting an array, finding the shortest path between two nodes on a graph, etc.). Hence, we evaluate whether fine-tuning can be used to enhance LLMs’ ability to compute stable matchings—a task requiring reasoning over ranked preferences in addition to algorithmic understanding. To this end, we perform LoRA fine-tuning [33] on three reasoning models, including Qwen-QwQ-32B and two smaller models, DeepSeek-8B and DeepSeek-14B, for Generating Stable Solutions task. Additionally, we also evaluate whether fine-tuning can mitigate errors made by LLMs in the Preference Reasoning task (Section 6).

Training. Let $\mathcal{D} = \{(\mathbf{x}^{(i)}, \mathbf{y}^{(i)})\}_{i=1}^N$ be the fine-tuning dataset for a given task. Each pair consists of an *input instance* $\mathbf{x}^{(i)}$ and the *desired model completion* $\mathbf{y}^{(i)}$. The input instance $\mathbf{x}^{(i)}$ is made up of four components: (i) a generic *system-prompt* s , (ii) a high-level *instruction* u , the *preference profile* $p^{(i)}$, and (iv) the *task-prompt* $t^{(i)}$. The desired completion $\mathbf{y}^{(i)}$ consists of two components, (i) a chain-of-thought *reasoning trace* $r^{(i)}$, and (ii) the *answer* $a^{(i)}$ in the desired format. Each model is fine-tuned with standard next-token cross-entropy on the concatenated sequence $\mathbf{z}^{(i)} = \mathbf{x}^{(i)} \parallel \mathbf{y}^{(i)}$. We separately fine-tune each model for the Generating Stable solutions task ($N = 10,000$) and the Preference Reasoning task ($N = 9,000$). See Appendix G for details of the fine-tuning setup.

Improvement. Fine-tuning LLMs with data containing synthetically generated reasoning traces substantially improves LLMs performance on both tasks, as evidenced in Table 2.

Table 3: The performance of LLMs with different reasoning capability across all tasks requiring reasoning over ranked preferences and executing structured algorithms.

Category	Model	Generating Stable Solutions			Resolving Instability			Detecting Instability	Preference Reasoning
		Stable Solutions (%)	Instability Rate (↓)	Optimality Rate (↑)	Stable Solutions (%)	Instability Rate (↓)	Optimality Rate (↑)	Accuracy (%)	Accuracy (%)
Basic	Llama-3.3-70B	0.33	54.03	0.21	0.17	56.04	0.25	56.76	52.67
	Gemini-2.0-Flash	2.36	48.44	0.21	0.83	53.94	0.22	58.38	58.89
Reasoning	DeepSeek-70B	26.20	23.43	0.59	22.49	24.61	0.60	77.05	88.67
	Qwen-QwQ-32B	35.67	21.22	0.63	28.00	26.37	0.62	75.05	96.89
Advanced Reasoning	o3-mini	58.00	19.98	0.72	57.50	18.52	0.75	86.67	98.78
	DeepSeek-R1	64.22	12.35	0.80	55.73	14.21	0.79	88.19	96.22
	Gemini-2.5-Pro	68.33	7.16	0.84	61.33	8.80	0.79	92.38	99.67

In fact, this approach enhances the performance of Qwen-QwQ-32B, a (non-advanced) reasoning model, to a success rate of 100% in computing stable matchings with *both* Easy and Medium instances, significantly outperforming advanced-reasoning models. Fine-tuning also clearly improves smaller models, i.e. DeepSeek-8B and DeepSeek-14B, both in terms of Correctness and Instability Rate.⁷

Similar results are obtained for the Preference Reasoning tasks, with the error-rate often reducing to 0 at the Easy and Medium levels.⁸

In spite of these improvements, however, there remains distinct gap in performance at the Easy and Medium levels as compared to the Hard level. LLMs remain altogether unable to compute stable matchings for Hard instances, even after fine-tuning. A similar trend is reflected in the accuracy on the Preference Reasoning task. Hence, while fine-tuning clearly improves the reasoning capabilities of LLMs, further enhancements are required to improve their ability to handle larger inputs.

Table 2: Improvement in performance in the Generating Stable Solutions and Preference Reasoning tasks after fine-tuning on respective datasets.

Model	Stage	Generating Stable Solutions						Preference Reasoning		
		Stable Solutions (%)			Instability Rate (↓)			Accuracy (%)		
		Easy	Med.	Hard	Easy	Med.	Hard	Easy	Med.	Hard
DeepSeek-8B	Vanilla	3.0	0.0	0.0	41.02	64.19	92.70	81.67	74.33	47.67
	Fine-tuned	55.0	44.0	0.0	11.40	20.25	—	100.0	98.33	75.00
DeepSeek-14B	Vanilla	19.0	0.0	0.0	20.66	55.31	94.09	97.67	91.33	72.33
	Fine-tuned	51.0	41.0	0.0	16.35	24.42	84.00	100.0	100.0	91.00
Qwen-QwQ-32B	Vanilla	83.0	24.0	0.0	2.35	19.27	59.07	99.00	100.0	91.67
	Fine-tuned	100.0	100.0	0.0	0.00	0.00	55.19	100.0	100.0	99.00

8 Concluding Remarks

We summarize the performance of LLMs across all four tasks in Table 3, reflecting the clear hierarchy between advanced-reasoning models, (non-advanced) reasoning models, and basic models. The limitations in reliably reasoning about ranked preferences raise concerns about the viability of LLMs as agents acting on behalf of users in market-oriented or preference-sensitive decision-making settings, limits their capacity to negotiate complex user preferences, and hinders efforts in developing pluralistic techniques (e.g., constitutional AI [2] and social choice-theoretic [15]) for value alignment that are inherently based on aggregating rankings.

Open-Source vs. Closed-Source models. Among the models that we evaluate, Gemini-2.5-Pro (a closed-source model) emerges as the most capable across all tasks. While DeepSeek-R1 (open-source) broadly outperforms OpenAI’s o3-mini (closed-source), it performs much worse with IC instances than with ML instances. While both basic models struggle on all tasks, Gemini-2.0-Flash (closed-source) marginally outperforms Llama-3.3-70B (open-source) on various metrics. Given the promising improvement yielded by fine-tuning an open-source reasoning model, i.e. Qwen-QwQ-32B, it is worth exploring strategies also that enable it to handle large inputs.

Beyond Linear Preferences. Our current evaluation paradigm considers complete and strict linear preferences. In real-world scenarios, however, preferences involve complexities such as incompleteness, indifference between alternatives, and capacity constraints [18, 43, 51]. As a result, algorithms to compute stable solutions in such settings are far more complex and solutions are often intractable [50]. A meaningful next step would be to examine how AI models respond to such situations.

⁷Interestingly, this improvement is clearer for ML instances, where both models achieve a 100% success rate at the Easy level and > 80% success rate at the Medium level.

⁸The only exception being DeepSeek-8B at the Medium level.

References

- [1] Hernan Abeledo and Uriel G Rothblum. Paths to marriage stability. *Discrete Applied Mathematics*, 63(1):1–12, 1995.
- [2] Yuntao Bai, Saurav Kadavath, Sandipan Kundu, Amanda Askell, Jackson Kernion, Andy Jones, Anna Chen, Anna Goldie, Azalia Mirhoseini, Cameron McKinnon, et al. Constitutional AI: Harmlessness from AI feedback. *arXiv preprint arXiv:2212.08073*, 2022.
- [3] Maciej Besta, Nils Blach, Ales Kubicek, Robert Gerstenberger, Michał Podstawski, Lukas Gianinazzi, Joanna Gajda, Tomasz Lehmann, Hubert Niewiadomski, Piotr Nyczyk, and Torsten Hoefer. Graph of thoughts: solving elaborate problems with large language models. In *Proceedings of the Thirty-Eighth AAAI Conference on Artificial Intelligence and Thirty-Sixth Conference on Innovative Applications of Artificial Intelligence and Fourteenth Symposium on Educational Advances in Artificial Intelligence*, AAAI’24/IAAI’24/EAAI’24. AAAI Press, 2024. ISBN 978-1-57735-887-9. doi: 10.1609/aaai.v38i16.29720. URL <https://doi.org/10.1609/aaai.v38i16.29720>.
- [4] Duncan Black et al. The theory of committees and elections. 1958.
- [5] Niclas Boehmer, Piotr Faliszewski, Łukasz Janeczko, Andrzej Kaczmarczyk, Grzegorz Lisowski, Grzegorz Pierczyński, Simon Rey, Dariusz Stolicki, Stanisław Szufa, and Tomasz Wąs. Guide to numerical experiments on elections in computational social choice. *arXiv preprint arXiv:2402.11765*, 2024.
- [6] Robert Brederick, Klaus Heeger, Dušan Knop, and Rolf Niedermeier. Multidimensional stable roommates with master list, 2021. URL <https://arxiv.org/abs/2009.14191>.
- [7] Angelina Brilliantova and Hadi Hosseini. Fair stable matching meets correlated preferences. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems*, pages 190–198, 2022.
- [8] Huanqia Cai, Yijun Yang, and Zhifeng Li. System-2 mathematical reasoning via enriched instruction tuning. *CoRR*, abs/2412.16964, 2024. doi: 10.48550/ARXIV.2412.16964. URL <https://doi.org/10.48550/arXiv.2412.16964>.
- [9] Ioannis Caragiannis and Karl Fehrs. Beyond the worst case: Distortion in impartial culture electorates, 2024. URL <https://arxiv.org/abs/2307.07350>.
- [10] Qiguang Chen, Libo Qin, Jinhao Liu, Dengyun Peng, Jiannan Guan, Peng Wang, Mengkang Hu, Yuhang Zhou, Te Gao, and Wanxiang Che. Towards reasoning era: A survey of long chain-of-thought for reasoning large language models. *CoRR*, abs/2503.09567, 2025. doi: 10.48550/ARXIV.2503.09567. URL <https://doi.org/10.48550/arXiv.2503.09567>.
- [11] Yiting Chen, Tracy Xiao Liu, You Shan, and Songfa Zhong. The emergence of economic rationality of gpt. *Proceedings of the National Academy of Sciences*, 120(51):e2316205120, 2023.
- [12] Haoang Chi, He Li, Wenjing Yang, Feng Liu, Long Lan, Xiaoguang Ren, Tongliang Liu, and Bo Han. Unveiling causal reasoning in large language models: Reality or mirage? In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural Information Processing Systems*, volume 37, pages 96640–96670. Curran Associates, Inc., 2024. URL https://proceedings.neurips.cc/paper_files/paper/2024/file/af2bb2b2280d36f8842e440b4e275152-Paper-Conference.pdf.
- [13] Hyung Won Chung, Le Hou, Shayne Longpre, Barret Zoph, Yi Tay, William Fedus, Yunxuan Li, Xuezhi Wang, Mostafa Dehghani, Siddhartha Brahma, Albert Webson, Shixiang Shane Gu, Zhuyun Dai, Mirac Suzgun, Xinyun Chen, Aakanksha Chowdhery, Alex Castro-Ros, Marie Pellat, Kevin Robinson, Dasha Valter, Sharan Narang, Gaurav Mishra, Adams Yu, Vincent Y. Zhao, Yanping Huang, Andrew M. Dai, Hongkun Yu, Slav Petrov, Ed H. Chi, Jeff Dean, Jacob Devlin, Adam Roberts, Denny Zhou, Quoc V. Le, and Jason Wei. Scaling instruction-finetuned language models. *J. Mach. Learn. Res.*, 25:70:1–70:53, 2024. URL <https://jmlr.org/papers/v25/23-0870.html>.

- [14] Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems. *CoRR*, abs/2110.14168, 2021. URL <https://arxiv.org/abs/2110.14168>.
- [15] Vincent Conitzer, Rachel Freedman, Jobst Heitzig, Wesley H Holliday, Bob M Jacobs, Nathan Lambert, Milan Mossé, Eric Pacuit, Stuart Russell, Hailey Schoelkopf, et al. Social choice should guide AI alignment in dealing with diverse human feedback. *arXiv preprint arXiv:2404.10271*, 2024.
- [16] Google Deepmind. Gemini pro, Mar 2025. URL <https://deepmind.google/technologies/gemini/pro/>.
- [17] DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, Aixin Liu, Bing Xue, Bingxuan Wang, Bochao Wu, Bei Feng, Chengda Lu, Chenggang Zhao, Chengqi Deng, Chenyu Zhang, Chong Ruan, Damai Dai, Deli Chen, Dongjie Ji, Erhang Li, Fangyun Lin, Fucong Dai, Fuli Luo, Guangbo Hao, Guanting Chen, Guowei Li, H. Zhang, Han Bao, Hanwei Xu, Haocheng Wang, Honghui Ding, Huajian Xin, Huazuo Gao, Hui Qu, Hui Li, Jianzhong Guo, Jiashi Li, Jiawei Wang, Jingchang Chen, Jingyang Yuan, Junjie Qiu, Junlong Li, J. L. Cai, Jiaqi Ni, Jian Liang, Jin Chen, Kai Dong, Kai Hu, Kaige Gao, Kang Guan, Kexin Huang, Kuai Yu, Lean Wang, Lecong Zhang, Liang Zhao, Litong Wang, Liyue Zhang, Lei Xu, Leyi Xia, Mingchuan Zhang, Minghua Zhang, Minghui Tang, Meng Li, Miaojun Wang, Mingming Li, Ning Tian, Panpan Huang, Peng Zhang, Qiancheng Wang, Qinyu Chen, Qiushi Du, Ruiqi Ge, Ruisong Zhang, Ruizhe Pan, Runji Wang, R. J. Chen, R. L. Jin, Ruyi Chen, Shanghao Lu, Shangyan Zhou, Shanhuang Chen, Shengfeng Ye, Shiyu Wang, Shuiping Yu, Shunfeng Zhou, Shuting Pan, and S. S. Li. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *CoRR*, abs/2501.12948, 2025. doi: 10.48550/ARXIV.2501.12948. URL <https://doi.org/10.48550/arXiv.2501.12948>.
- [18] Maxence Delorme, Sergio García, Jacek Gondzio, Jörg Kalcsics, David F. Manlove, and William Petteersson. Mathematical models for stable matching problems with ties and incomplete lists. *Eur. J. Oper. Res.*, 277(2):426–441, 2019. doi: 10.1016/J.EJOR.2019.03.017. URL <https://doi.org/10.1016/j.ejor.2019.03.017>.
- [19] Qingxiu Dong, Lei Li, Damai Dai, Ce Zheng, Jingyuan Ma, Rui Li, Heming Xia, Jingjing Xu, Zhiyong Wu, Tianyu Liu, et al. A survey on in-context learning. *arXiv preprint arXiv:2301.00234*, 2022.
- [20] Jinhao Duan, Renming Zhang, James Diffenderfer, Bhavya Kailkhura, Lichao Sun, Elias Stengel-Eskin, Mohit Bansal, Tianlong Chen, and Kaidi Xu. Gtbench: Uncovering the strategic reasoning capabilities of llms via game-theoretic evaluations. In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural Information Processing Systems*, volume 37, pages 28219–28253. Curran Associates, Inc., 2024. URL https://proceedings.neurips.cc/paper_files/paper/2024/file/3191170938b6102e5c203b036b7c16dd-Paper-Conference.pdf.
- [21] Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, Aurélien Rodriguez, Austen Gregerson, Ava Spataru, Baptiste Rozière, Bethany Biron, Binh Tang, Bobbie Chern, Charlotte Caucheteux, Chaya Nayak, Chloe Bi, Chris Marra, Chris McConnell, Christian Keller, Christophe Touret, Chunyang Wu, Corinne Wong, Cristian Canton Ferrer, Cyrus Nikolaidis, Damien Allonsius, Daniel Song, Danielle Pintz, Danny Livshits, David Esiobu, Dhruv Choudhary, Dhruv Mahajan, Diego Garcia-Olano, Diego Perino, Dieuwke Hupkes, Egor Lakomkin, Ehab AlBadawy, Elina Lobanova, Emily Dinan, Eric Michael Smith, Filip Radenovic, Frank Zhang, Gabriel Synnaeve, Gabrielle Lee, Georgia Lewis Anderson, Graeme Nail, Grégoire Mialon, Guan Pang, Guillem Cucurell, Hailey Nguyen, Hannah Korevaar, Hu Xu, Hugo Touvron, Iliyan Zarov, Imanol Arrieta Ibarra, Isabel M. Kloumann, Ishan Misra, Ivan Evtimov, Jade Copet, Jaewon Lee, Jan Geffert, Jana Vranes, Jason

- 465 Park, Jay Mahadeokar, Jeet Shah, Jelmer van der Linde, Jennifer Billock, Jenny Hong, Jenya Lee,
466 Jeremy Fu, Jianfeng Chi, Jianyu Huang, Jiawen Liu, Jie Wang, Jiecao Yu, Joanna Bitton, Joe
467 Spisak, Jongsoo Park, Joseph Rocca, Joshua Johnstun, Joshua Saxe, Junteng Jia, Kalyan Vasuden
468 Alwala, Kartikeya Upasani, Kate Plawiak, Ke Li, Kenneth Heafield, Kevin Stone, and et al.
469 The llama 3 herd of models. *CoRR*, abs/2407.21783, 2024. doi: 10.48550/ARXIV.2407.21783.
470 URL <https://doi.org/10.48550/arXiv.2407.21783>.
- 471 [22] Ömer Eğecioğlu and Ayça E Giritligil. The impartial, anonymous, and neutral culture model:
472 a probability model for sampling public preference structures. *The Journal of Mathematical*
473 *Sociology*, 37(4):203–222, 2013.
- 474 [23] Sara Fish, Paul Gözl, David C. Parkes, Ariel D. Procaccia, Gili Rusak, Itai Shapira, and Manuel
475 Wüthrich. Generative social choice. In Dirk Bergemann, Robert Kleinberg, and Daniela Sabán,
476 editors, *Proceedings of the 25th ACM Conference on Economics and Computation, EC 2024,*
477 *New Haven, CT, USA, July 8-11, 2024*, page 985. ACM, 2024. doi: 10.1145/3670865.3673547.
478 URL <https://doi.org/10.1145/3670865.3673547>.
- 479 [24] Sara Fish, Julia Shephard, Minkai Li, Ran I. Shorrer, and Yannai A. Gonczarowski. Econevals:
480 Benchmarks and litmus tests for LLM agents in unknown environments. *CoRR*, abs/2503.18825,
481 2025. doi: 10.48550/ARXIV.2503.18825. URL [https://doi.org/10.48550/arXiv.2503.](https://doi.org/10.48550/arXiv.2503.18825)
482 18825.
- 483 [25] R. A. Fisher. On the interpretation of χ^2 from contingency tables, and the calculation of p.
484 *Journal of the Royal Statistical Society*, 85(1):87–94, 1922. ISSN 09528385. URL [http:](http://www.jstor.org/stable/2340521)
485 [//www.jstor.org/stable/2340521](http://www.jstor.org/stable/2340521).
- 486 [26] David Gale and Lloyd S Shapley. College admissions and the stability of marriage. *The*
487 *American Mathematical Monthly*, 69(1):9–15, 1962.
- 488 [27] Chengguang Gan, Qinghao Zhang, and Tatsunori Mori. Application of LLM agents in recruit-
489 ment: A novel framework for automated resume screening. *Journal of Information Processing*,
490 32:881–893, 2024.
- 491 [28] Jiayi Gui, Yiming Liu, Jiale Cheng, Xiaotao Gu, Xiao Liu, Hongning Wang, Yuxiao Dong, Jie
492 Tang, and Minlie Huang. Logicgame: Benchmarking rule-based reasoning abilities of large
493 language models. *CoRR*, abs/2408.15778, 2024. doi: 10.48550/ARXIV.2408.15778. URL
494 <https://doi.org/10.48550/arXiv.2408.15778>.
- 495 [29] Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and
496 Jacob Steinhardt. Measuring massive multitask language understanding. In *9th International*
497 *Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*.
498 OpenReview.net, 2021. URL <https://openreview.net/forum?id=d7KBjmI3GmQ>.
- 499 [30] Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric
500 Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving
501 with the MATH dataset. In Joaquin Vanschoren and Sai-Kit Yeung, editors, *Pro-*
502 *ceedings of the Neural Information Processing Systems Track on Datasets and Bench-*
503 *marks 1, NeurIPS Datasets and Benchmarks 2021, December 2021, virtual*, 2021.
504 URL [https://datasets-benchmarks-proceedings.neurips.cc/paper/2021/hash/](https://datasets-benchmarks-proceedings.neurips.cc/paper/2021/hash/be83ab3ecd0db773eb2dc1b0a17836a1-Abstract-round2.html)
505 [be83ab3ecd0db773eb2dc1b0a17836a1-Abstract-round2.html](https://datasets-benchmarks-proceedings.neurips.cc/paper/2021/hash/be83ab3ecd0db773eb2dc1b0a17836a1-Abstract-round2.html).
- 506 [31] John J Horton. Large language models as simulated economic agents: What can we learn from
507 homo silicus? Technical report, National Bureau of Economic Research, 2023.
- 508 [32] Hadi Hosseini and Samarth Khanna. Distributive fairness in large language models: Evaluating
509 alignment with human values. *arXiv preprint arXiv:2502.00313*, 2025.
- 510 [33] Edward J. Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yanzhi Li, Shean Wang,
511 Lu Wang, and Weizhu Chen. Lora: Low-rank adaptation of large language models. In *The Tenth*
512 *International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29,*
513 *2022*. OpenReview.net, 2022. URL <https://openreview.net/forum?id=nZeVKeeFYf9>.

- [34] David Huang, Francisco J. Marmolejo Cossío, Edwin Lock, and David C. Parkes. Accelerated preference elicitation with llm-based proxies. *CoRR*, abs/2501.14625, 2025. doi: 10.48550/ARXIV.2501.14625. URL <https://doi.org/10.48550/arXiv.2501.14625>.
- [35] Robert W Irving, David F Manlove, and Sandy Scott. The stable marriage problem with master preference lists. *Discrete Applied Mathematics*, 156(15):2959–2977, 2008.
- [36] Jingru Jia and Zehua Yuan. An experimental study of competitive market behavior through llms. *arXiv preprint arXiv:2409.08357*, 2024.
- [37] Jingru Jia, Zehua Yuan, Junhao Pan, Paul McNamara, and Deming Chen. Large language model strategic reasoning evaluation through behavioral game theory. *CoRR*, abs/2502.20432, 2025. doi: 10.48550/ARXIV.2502.20432. URL <https://doi.org/10.48550/arXiv.2502.20432>.
- [38] Carlos E. Jimenez, John Yang, Alexander Wettig, Shunyu Yao, Kexin Pei, Ofir Press, and Karthik R. Narasimhan. Swe-bench: Can language models resolve real-world github issues? In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024. URL <https://openreview.net/forum?id=VTF8yNQM66>.
- [39] Enric Junqué De Fortuny. Simulating market equilibrium with large language models. 2025.
- [40] Naoyuki Kamiyama. Stable matchings with ties, master preference lists, and matroid constraints. In Martin Hoefer, editor, *Algorithmic Game Theory*, pages 3–14, Berlin, Heidelberg, 2015. Springer Berlin Heidelberg. ISBN 978-3-662-48433-3.
- [41] Donald Ervin Knuth. Marriages stables. *Technical report*, 1976.
- [42] Donald Ervin Knuth. *Stable marriage and its relation to other combinatorial problems: An introduction to the mathematical analysis of algorithms*, volume 10. American Mathematical Soc., 1997.
- [43] Augustine Kwanashie and David F. Manlove. An integer programming approach to the hospitals/residents problem with ties. In Dennis Huisman, Ilse Louwerse, and Albert P. M. Wagelmans, editors, *Operations Research Proceedings 2013, Selected Papers of the International Conference on Operations Research, OR2013, organized by the German Operations Research Society (GOR), the Dutch Society of Operations Research (NGB) and Erasmus University Rotterdam, September 3-6, 2013*, pages 263–269. Springer, 2013. doi: 10.1007/978-3-319-07001-8_36. URL https://doi.org/10.1007/978-3-319-07001-8_36.
- [44] Aitor Lewkowycz, Anders Andreassen, David Dohan, Ethan Dyer, Henryk Michalewski, Vinay V. Ramasesh, Ambrose Slone, Cem Anil, Imanol Schlag, Theo Gutman-Solo, Yuhuai Wu, Behnam Neyshabur, Guy Gur-Ari, and Vedant Misra. Solving quantitative reasoning problems with language models. In Sanmi Koyejo, S. Mohamed, A. Agarwal, Danielle Belgrave, K. Cho, and A. Oh, editors, *Advances in Neural Information Processing Systems 35: Annual Conference on Neural Information Processing Systems 2022, NeurIPS 2022, New Orleans, LA, USA, November 28 - December 9, 2022*, 2022. URL http://papers.nips.cc/paper_files/paper/2022/hash/18abbef8cfe9203fdf9053c9c4fe191-Abstract-Conference.html.
- [45] Chengpeng Li, Zheng Yuan, Hongyi Yuan, Guanting Dong, Keming Lu, Jiancan Wu, Chuanqi Tan, Xiang Wang, and Chang Zhou. Mugglemath: Assessing the impact of query and response augmentation on math reasoning. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar, editors, *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), ACL 2024, Bangkok, Thailand, August 11-16, 2024*, pages 10230–10258. Association for Computational Linguistics, 2024. doi: 10.18653/V1/2024.ACL-LONG.551. URL <https://doi.org/10.18653/v1/2024.acl-long.551>.
- [46] Zhong-Zhi Li, Duzhen Zhang, Ming-Liang Zhang, Jiaxin Zhang, Zengyan Liu, Yuxuan Yao, Haotian Xu, Junhao Zheng, Pei-Jie Wang, Xiuyi Chen, Yingying Zhang, Fei Yin, Jiahua Dong, Zhijiang Guo, Le Song, and Cheng-Lin Liu. From system 1 to system 2: A survey of reasoning large language models. *CoRR*, abs/2502.17419, 2025. doi: 10.48550/ARXIV.2502.17419. URL <https://doi.org/10.48550/arXiv.2502.17419>.

- [47] Yinhong Liu, Zhijiang Guo, Tianya Liang, Ehsan Shareghi, Ivan Vulic, and Nigel Collier. Aligning with logic: Measuring, evaluating and improving logical consistency in large language models. *CoRR*, abs/2410.02205, 2024. doi: 10.48550/ARXIV.2410.02205. URL <https://doi.org/10.48550/arXiv.2410.02205>.
- [48] Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu, Lei Meng, Jiao Sun, and Abhinav Rastogi. Improve mathematical reasoning in language models by automated process supervision. *CoRR*, abs/2406.06592, 2024. doi: 10.48550/ARXIV.2406.06592. URL <https://doi.org/10.48550/arXiv.2406.06592>.
- [49] Ziyang Luo, Can Xu, Pu Zhao, Qingfeng Sun, Xiubo Geng, Wenxiang Hu, Chongyang Tao, Jing Ma, Qingwei Lin, and Daxin Jiang. Wizardcoder: Empowering code large language models with evol-instruct. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net, 2024. URL <https://openreview.net/forum?id=UnUwSIgK5W>.
- [50] David F. Manlove. Algorithmics of matching under preferences. *Bull. EATCS*, 112, 2014. URL <http://eatcs.org/beatcs/index.php/beatcs/article/view/252>.
- [51] David F. Manlove, Duncan Milne, and Sofiat Olaosebikan. Student-project allocation with preferences over projects: Algorithmic and experimental results. *Discret. Appl. Math.*, 308: 220–234, 2022. doi: 10.1016/J.DAM.2020.08.015. URL <https://doi.org/10.1016/j.dam.2020.08.015>.
- [52] Larisa Markeeva, Sean McLeish, Borja Ibarz, Wilfried Bounsi, Olga Kozlova, Alex Vitvitskyi, Charles Blundell, Tom Goldstein, Avi Schwarzschild, and Petar Velickovic. The clrs-text algorithmic reasoning language benchmark. *CoRR*, abs/2406.04229, 2024. doi: 10.48550/ARXIV.2406.04229. URL <https://doi.org/10.48550/arXiv.2406.04229>.
- [53] David G McVitie and Leslie B Wilson. The stable marriage problem. *Communications of the ACM*, 14(7):486–490, 1971.
- [54] Seyed-Iman Mirzadeh, Keivan Alizadeh, Hooman Shahrokhi, Oncel Tuzel, Samy Bengio, and Mehrdad Farajtabar. Gsm-symbolic: Understanding the limitations of mathematical reasoning in large language models. *CoRR*, abs/2410.05229, 2024. doi: 10.48550/ARXIV.2410.05229. URL <https://doi.org/10.48550/arXiv.2410.05229>.
- [55] Terufumi Morishita, Gaku Morio, Atsuki Yamaguchi, and Yasuhiro Sogawa. Enhancing reasoning capabilities of llms via principled synthetic logic corpus. In Amir Globersons, Lester Mackey, Danielle Belgrave, Angela Fan, Ulrich Paquet, Jakub M. Tomczak, and Cheng Zhang, editors, *Advances in Neural Information Processing Systems 38: Annual Conference on Neural Information Processing Systems 2024, NeurIPS 2024, Vancouver, BC, Canada, December 10 - 15, 2024*, 2024. URL http://papers.nips.cc/paper_files/paper/2024/hash/8678da90126aa58326b2fc0254b33a8c-Abstract-Conference.html.
- [56] OpenAI. Openai o3-mini system card, 2025. URL <https://openai.com/index/o3-mini-system-card/>.
- [57] Sundar Pichai, Demis Hassabis, and Kouray Kavukcuoglu. Introducing gemini 2.0: our new ai model for the agentic era, Dec 2024. URL <https://blog.google/technology/google-deepmind/google-gemini-ai-update-december-2024/>.
- [58] Alvin E Roth and John H Vande Vate. Random paths to stability in two-sided matching. *Econometrica: Journal of the Econometric Society*, pages 1475–1480, 1990.
- [59] Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. Deepseekmath: Pushing the limits of mathematical reasoning in open language models, 2024. URL <https://arxiv.org/abs/2402.03300>.
- [60] Kiwon Song, James M Jennings III, and Clinton P Davis-Stober. Benchmarking the rationality of AI decision making using the transitivity axiom. *arXiv preprint arXiv:2502.10554*, 2025.

- [61] Ermis Soumalias, Yanchen Jiang, Kehang Zhu, Michael J. Curry, Sven Seuken, and David C. Parkes. Llm-powered preference elicitation in combinatorial assignment. *CoRR*, abs/2502.10308, 2025. doi: 10.48550/ARXIV.2502.10308. URL <https://doi.org/10.48550/arXiv.2502.10308>.
- [62] Wenjie Tang, Yuan Zhou, Erqiang Xu, Keyan Cheng, Minne Li, and Liquan Xiao. Dsgbench: A diverse strategic game benchmark for evaluating llm-based agents in complex decision-making environments. *CoRR*, abs/2503.06047, 2025. doi: 10.48550/ARXIV.2503.06047. URL <https://doi.org/10.48550/arXiv.2503.06047>.
- [63] Qwen Team. Qwq: Reflect deeply on the boundaries of the unknown, Nov 2024. URL <https://qwenlm.github.io/blog/qwq-32b-preview/>.
- [64] Ye Tian, Baolin Peng, Linfeng Song, Lifeng Jin, Dian Yu, Lei Han, Haitao Mi, and Dong Yu. Toward self-improvement of llms via imagination, searching, and criticizing. In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural Information Processing Systems*, volume 37, pages 52723–52748. Curran Associates, Inc., 2024. URL https://proceedings.neurips.cc/paper_files/paper/2024/file/5e5853f35164e434015716a8c2a66543-Paper-Conference.pdf.
- [65] Vince Trencsenyi, Agnieszka Mensfelt, and Kostas Stathis. Approximating human strategic reasoning with llm-enhanced recursive reasoners leveraging multi-agent hypergames. *CoRR*, abs/2502.07443, 2025. doi: 10.48550/ARXIV.2502.07443. URL <https://doi.org/10.48550/arXiv.2502.07443>.
- [66] Ilia Tsetlin, Michel Regenwetter, and Bernard Grofman. The impartial culture maximizes the probability of majority cycles. *Social Choice and Welfare*, 21(3):387–398, 2003. ISSN 01761714, 1432217X. URL <http://www.jstor.org/stable/41106568>.
- [67] Emily Vaillancourt and Christopher Thompson. Instruction tuning on large language models to improve reasoning performance. June 2024. doi: 10.36227/techrxiv.171778765.53860614/v1. URL <http://dx.doi.org/10.36227/techrxiv.171778765.53860614/v1>.
- [68] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35:24824–24837, 2022.
- [69] B. L. Welch. The generalization of ‘student’s’ problem when several different population variances are involved. *Biometrika*, 34(1/2):28–35, 1947. ISSN 00063444. URL <http://www.jstor.org/stable/2332510>.
- [70] Kai Xiong, Xiao Ding, Ting Liu, Bing Qin, Dongliang Xu, Qing Yang, Hongtao Liu, and Yixin Cao. Meaningful learning: Enhancing abstract reasoning in large language models via generic fact guidance. In Amir Globersons, Lester Mackey, Danielle Belgrave, Angela Fan, Ulrich Paquet, Jakub M. Tomczak, and Cheng Zhang, editors, *Advances in Neural Information Processing Systems 38: Annual Conference on Neural Information Processing Systems 2024, NeurIPS 2024, Vancouver, BC, Canada, December 10 - 15, 2024*, 2024. URL http://papers.nips.cc/paper_files/paper/2024/hash/da5498f88193ff61f0daea1940b819da-Abstract-Conference.html.
- [71] Yi Xu, Laura Ruis, Tim Rocktäschel, and Robert Kirk. Investigating non-transitivity in LLM-as-a-judge. *arXiv preprint arXiv:2502.14074*, 2025.
- [72] Joshua C. Yang, Damian Dailisan, Marcin Korecki, Carina I. Hausladen, and Dirk Helbing. LLM voting: Human choices and AI collective decision-making. In Sanmay Das, Brian Patrick Green, Kush Varshney, Marianna Ganapini, and Andrea Renda, editors, *Proceedings of the Seventh AAAI/ACM Conference on AI, Ethics, and Society (AIES-24) - Full Archival Papers, October 21-23, 2024, San Jose, California, USA - Volume 1*, pages 1696–1708. AAAI Press, 2024. doi: 10.1609/AIES.V7I1.31758. URL <https://doi.org/10.1609/aies.v7i1.31758>.
- [73] Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: deliberate problem solving with large language models. In *Proceedings of the 37th International Conference on Neural Information Processing Systems, NIPS ’23*, Red Hook, NY, USA, 2023. Curran Associates Inc.

- 666 [74] Xuan Zhang, Chao Du, Tianyu Pang, Qian Liu, Wei Gao, and Min Lin. Chain of pref-
 667 erence optimization: Improving chain-of-thought reasoning in llms. In A. Globerson,
 668 L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances*
 669 *in Neural Information Processing Systems*, volume 37, pages 333–356. Curran Associates,
 670 Inc., 2024. URL [https://proceedings.neurips.cc/paper_files/paper/2024/file/](https://proceedings.neurips.cc/paper_files/paper/2024/file/00d80722b756de0166523a87805dd00f-Paper-Conference.pdf)
 671 [00d80722b756de0166523a87805dd00f-Paper-Conference.pdf](https://proceedings.neurips.cc/paper_files/paper/2024/file/00d80722b756de0166523a87805dd00f-Paper-Conference.pdf).
- 672 [75] Heng Zhou, Hejia Geng, Xiangyuan Xue, Zhenfei Yin, and Lei Bai. Reso: A reward-driven self-
 673 organizing llm-based multi-agent system for reasoning tasks. *CoRR*, abs/2503.02390, 2025. doi:
 674 10.48550/ARXIV.2503.02390. URL <https://doi.org/10.48550/arXiv.2503.02390>.
- 675 [76] Pei Zhou, Jay Pujara, Xiang Ren, Xinyun Chen, Heng-Tze Cheng, Quoc V. Le, Ed H.,
 676 Denny Zhou, Swaroop Mishra, and Huaixiu Steven Zheng. Self-discover: Large lan-
 677 guage models self-compose reasoning structures. In A. Globerson, L. Mackey, D. Bel-
 678 grave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural*
 679 *Information Processing Systems*, volume 37, pages 126032–126058. Curran Associates,
 680 Inc., 2024. URL [https://proceedings.neurips.cc/paper_files/paper/2024/file/](https://proceedings.neurips.cc/paper_files/paper/2024/file/e41efb03e20ca3c231940a3c6917ef6f-Paper-Conference.pdf)
 681 [e41efb03e20ca3c231940a3c6917ef6f-Paper-Conference.pdf](https://proceedings.neurips.cc/paper_files/paper/2024/file/e41efb03e20ca3c231940a3c6917ef6f-Paper-Conference.pdf).

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1015 A Limitations and Future Work

1016 While we present a comprehensive evaluation of the practical algorithmic and economic reasoning
1017 capabilities of a series of state-of-the-art LLMs, our dataset primarily relies on synthetic data due
1018 to the challenges in obtaining real-world ordinal preference data. This calls for the collection and
1019 curation of datasets in the two-sided matching setting, and generating preference profile datasets that
1020 are better aligned with human preferences.

1021 Additionally, while our work provides insights into the reasons behind the failure of LLMs to
1022 consistently provide stable solutions (see Section 6), there is scope for further clarity on where exactly
1023 LLMs make mistakes during algorithmic execution. A potential method to explore this is to break the
1024 algorithmic execution task into smaller steps (e.g. a single proposal-acceptance/rejection cycle) and
1025 identify which components of the *state-transition* are challenging for LLMs to understand.

1026 Furthermore, while fine-tuning substantially enhances LLMs’ performance on instances with rela-
1027 tively smaller input sizes, improving their performance with larger inputs requires further exploration.
1028 This can include the investigation of methods such as fine-tuning the entire set of parameters (unlike
1029 with LoRA) or reinforcement-learning methods such as *group-relative policy optimization* (GRPO)
1030 that are known to increase the reasoning capabilities of LLMs [59].

1031 B Broader Impacts

1032 This paper is to advance Machine Learning and AI research, with a special emphasis on the reasoning
1033 ability of LLMs—an essential component of autonomous AI. We identify key shortcomings in the
1034 reasoning capabilities of LLMs, especially in terms of aggregating individuals preferences over
1035 alternatives and algorithmic execution. We believe that the findings in this work can inform further
1036 research into AI systems to enhance their ability to act independently in complex decision-making
1037 scenarios.

1038 C Preference-Based Algorithms in Matching Markets

1039 C.1 Algorithm for Generating a Stable Matching

1040 As shown in Algorithm 1 standard deferred-acceptance algorithm runs by having the proposing side
1041 of the market make a series of proposals, and each agent that receives at least one proposal decides
1042 which proposal to accept (the proposal becomes an engagement), and which proposals to reject (or
1043 engagements to break). If a proposal is rejected or an engagement is broken, the proposer and receiver
1044 remove each other from their preference lists. This continues until all agents are matched—the
1045 resulting solution is stable.

Algorithm 1 The Deferred-Acceptance Algorithm

```
assign each agent  $m \in M$  and  $w \in W$  to be free
while some man  $m$  is free do
   $w$  = first woman on  $m$ ’s preference list
   $m$  proposes and becomes engaged to  $w$ 
  if some man  $p$  is engaged to  $w$  then
    break the engagement  $(p, w)$ , assign  $p$  to be free
  end if
  for each  $m'$  in  $w$ ’s preference list s.t.  $wr(w, m) \succ wr(w, m')$  do
    remove  $m'$  and  $w$  from each other’s preferences
  end for
end while
output the list of engaged pairs, these form a stable matching
```

1046 C.2 Algorithm for Generating a Stable Matching w/ Master Lists

1047 Similar to Algorithm 1, Algorithm 2 runs in rounds of proposals. Due to the master lists preferences
1048 on the proposal-receiving side of the market, proposals are automatically accepted. If an agent m' on

1049 the proposing side is less preferred than a particular agent w 's current partner m , then m' and w are
 1050 removed from each other's preference lists.

Algorithm 2 The Deferred-Acceptance Algorithm for Preferences w/ Master Lists on One Side

```

assign each agent  $m \in M$  and  $w \in W$  to be free
while some man  $m$  is free do
   $w$  = first woman on  $m$ 's preference list
   $m$  proposes and becomes engaged to  $w$ 
  for each  $m'$  in  $w$ 's preference list s.t.  $wr(w, m) \succ wr(w, m')$  do
    remove  $m'$  and  $w$  from each other's preferences
  end for
end while
output the list of engaged pairs, these form a stable matching

```

1051 **C.3 Algorithm for Resolving Instability**

1052 While we don't explicitly describe the algorithm here, the mechanism presented by Abeledo and
 1053 Rothblum [1] can be applied to an unstable matching μ by resolving blocking pairs, resulting in a
 1054 stable solution. Intuitively, an LLM does not have to follow a specific mechanism, rather the model
 1055 can resolve instability by iteratively resolving blocking pairs as they arise (eventually, assuming all
 1056 steps are correct, the model should arrive at a stable solution).

1057 **C.4 Algorithm for Detecting Instability**

1058 Intuitively, Algorithm 3 works by iteratively visiting each pair of agents (m, w) s.t. $m \in M$ and
 1059 $w \in W$, and finding a pair such that either m prefers w to their current partner, or w prefers m to
 1060 their current partner (when such a pair is found, output it as a blocking pair). If no pair (m, w) is
 1061 found to be a blocking pair, then the solution is stable.

Algorithm 3 Stability Detecting Algorithm

```

for each  $(m, w) \in \mu$ , where  $m \in M$  and  $w \in W$ 
  for man  $m \in M$  do
    for man  $w \in W$  do
      if  $wr(w, m) \succ wr(w, \mu(w))$  and  $mr(m, w) \succ mr(m, \mu(w))$  then
        output the identified blocking pair  $(m, w)$ 
      end if
    end for
  end for
output that there exist no blocking pairs

```

1062 **D Prompt Engineering**

1063 **Providing Algorithmic Description.** In Table 4, we compare LLMs' performance when provided
 1064 with the pseudo-code for the DA algorithm as part of the prompt, to the case when no algorithm is
 1065 provided. While providing the DA algorithm as part of the prompt leads to an improvement in some
 1066 cases, the only case in which there is improvement is statistically significant⁹ is with DeepSeek-70B
 1067 at the Easy level with ML instances.

1068 **Reasoning-enhancement Prompts.** For models that fail at generating stable matchings even with
 1069 small instances, we evaluate whether prompt-based enhancements such as Chain-of-Thought (CoT)
 1070 [68] and Few-shot prompting [19] can improve their performance. In particular, we introduce the
 1071 following three types of modifications to the original prompt (used in Section 3):

⁹At $p < 0.05$, using Fisher's exact test

Table 4: Correctness (percentage of stable solutions) of solutions returned by LLMs when provided with the DA algorithm in the prompt (With) as compared to the case when not provided (Without).

Difficulty	Preference	Basic LLMs				Reasoning LLMs				Advanced Reasoning LLMs					
		Gemini-2.0-Flash		Llama-3.3-70B		Qwen-QwQ-32B		DeepSeek-70B		o3-mini		DeepSeek-R1		Gemini-2.5-Pro	
		Without	With	Without	With	Without	With	Without	With	Without	With	Without	With	Without	With
Easy	IC	6	10	2	0	76	84	70	74	100	98	100	96	98	100
	ML	8	6	2	6	90	88	72	94	96	100	98	100	98	98
Medium	IC	0	0	0	0	14	2	0	0	68	64	42	44	90	88
	ML	0	0	0	0	34	40	14	12	80	86	86	82	88	94
Hard	IC	0	0	0	0	0	0	0	0	0	0	0	0	8	6
	ML	0	0	0	0	0	0	0	0	0	0	54	36	40	38
Average		2.33	2.67	0.67	1.00	35.67	35.67	26.00	30.00	57.33	58.00	63.33	59.66	68.33	70.66

Table 5: Correctness (percentage of stable solutions) of solutions returned when prompt-enhancement strategies are used, as compared to the case without, for the Easy and Medium difficulty levels.¹⁰

Size	Model	Culture	Gemini-2.0-Flash				Llama-3.3-70B				Qwen-QwQ-32B				DeepSeek-70B			
			None	CoT-V	CoT-SL	Few-shot	None	CoT-V	CoT-SL	Few-shot	None	CoT-V	CoT-SL	Few-shot	None	CoT-V	CoT-SL	Few-shot
			IC	ML	IC	ML	IC	ML	IC	ML	IC	ML	IC	ML	IC	ML	IC	ML
10	IC	ML	6	2	0	0	2	0	0	0	76	86	84	90	70	60	60	68
			8	4	4	2	2	0	10	6	90	94	92	94	72	76	68	76
20	IC	ML	0	0	0	0	0	0	0	0	14	2	6	6	0	0	0	0
			0	0	0	0	0	0	0	0	34	36	34	42	14	10	4	2

- **CoT-Vanilla (CoT-V):** The steps of execution of the DA algorithm are provided for an example instance. Each step consists of a single (free) man making his next proposal, and the woman either accepting or rejecting the proposal based on her current status.
- **CoT-Shortlist (CoT-SL):** The steps involved in a modified way to execute the DA algorithm are provided. In each step, all men that the woman prefers less than the proposing man are deleted from her preference list, and the woman is deleted from the preference lists of the same set of men (since they would never be matched). This approach ensures that men never propose to women who will reject them—reducing the overall number of steps.
- **Few-shot Examples:** As opposed to the previous two cases, we provide LLMs with three examples of stable matching instances accompanied by their solutions.

To limit the context size of the prompt, we consider examples with $n = 5$ for each of these prompt modifications.

As depicted in Table 5, these prompting enhancements fail to improve the ability of LLMs to generate stable matchings. While there is a slight improvement for models like Qwen-QwQ-32B and DeepSeek-70B instances with Master-list preferences and size $n = 10$, this improvement is not statistically significant.

Modified Problem Setting. To measure whether LLMs are sensitive to the nomenclature used to described the two-sided matching market, we consider a different setting, i.e. that where workers are to be assigned tasks (and members on both sides have preferences over members of the other). We test the difference in the performance of two LLMs—Gemini-2.0-Flash and o3-mini—between the task-scheduling and stable-marriage settings. The results are provided in Table 6. While there is a slight decrease in performance for o3-mini, the change is not significant (at $p < 0.05$). This provides further evidence that LLMs understand requirements of computing stable solutions for matching markets, in general.

E Resolving Instability: Additional Material

E.1 Generating Unstable Matchings

Here we describe the procedures we use to generate the two types of unstable matchings we consider.

One-BP. We generate a matching that contains a single blocking pair, by swapping the partners of two randomly selected men in the Optimal matching. Since such a swap may lead to more than one

¹⁰All models considered here are never able to generate stable matchings at the Hard difficulty level, with any prompting method.

Table 6: Correctness of responses from two LLMs when the task is framed as the *stable marriage* and the *task-scheduling problem*.

Difficulty	Model	Gemini-2.0-F		o3-mini	
	Preference	Stable Matching	Task Scheduling	Stable Matching	Task Scheduling
10	IC	6	8	100	98
	ML	8	6	68	50
20	IC	0	0	0	0
	ML	0	0	100	98
50	IC	0	0	80	72
	ML	0	0	0	0

1101 blocking-pair (or no blocking pairs), we perform this process (for every instance) until we obtain a
 1102 matching with exactly one blocking-pair. This procedure is formally described in Algorithm 4.

Algorithm 4 GENERATEONEBPMATCHING

Require:

$\Pi = (\succ_m, \succ_w)$ \triangleright Preference profile for all men $m \in M$ and women $w \in W$
 μ^* \triangleright Men-optimal stable matching returned by Deferred Acceptance

Ensure:

A matching μ containing *exactly one* blocking pair

```

1: function GENERATEONEBPMATCHING( $\Pi, \mu^*$ )
2:   repeat  $\triangleright$  Keep trying until the condition is met
3:      $\mu \leftarrow \text{copy}(\mu^*)$   $\triangleright$  Start from the stable matching
4:      $(m_a, m_b) \leftarrow$  arbitrary pair  $m_a, m_b \in M$  s.t.  $m_a \neq m_b$ 
5:      $w_a \leftarrow \mu(m_a)$ 
6:      $w_b \leftarrow \mu(m_b)$   $\triangleright$  Swap partners of the two men
7:      $\mu(m_a) \leftarrow w_b, \mu(w_b) \leftarrow m_a$ 
8:      $\mu(m_b) \leftarrow w_a, \mu(w_a) \leftarrow m_b$ 
9:   until  $|\text{BLOCKINGPAIRS}(\mu, \Pi)| = 1$   $\triangleright$  Stop when exactly one blocking pair exists
10:  return  $\mu$ 
11: end function

12: function BLOCKINGPAIRS( $\mu, \Pi$ )
13:   $B \leftarrow \emptyset$ 
14:  for all  $m \in M$  do
15:    for all  $w \in W$  do
16:      if  $w \succ_m \mu(m)$  and  $m \succ_w \mu(w)$  then
17:         $B \leftarrow B \cup \{(m, w)\}$ 
18:      end if
19:    end for
20:  end for
21:  return  $B$ 
22: end function

```

1103 **Random.** A Random matching is simply generated by generating a random permutation of agents
 1104 on one side and assigning agents such a list to the agents on the other side, one-by-one.

1105 E.2 Further Results

1106 **Measuring Instability after Repair.** The extent to which a given matching is incorrect does
 1107 influence quality of the solutions returned by LLMs after being asked to correct it. Table 7 shows that
 1108 the number of blocking pairs is often significantly lower when the unstable matching (that LLMs
 1109 are asked to correct) has a single blocking pair, as compared to the case with random matchings.¹²
 1110 Similarly, as shown in Figure 6, matchings returned after resolving an almost stable matching have a
 1111 greater overlap with the Optimal solution (especially at Medium and Hard difficulty levels).

¹¹All statistical comparisons in this table are made using Welch’s t-test [69].

¹²In fact, asking LLMs to resolve a random matching leads to a significantly higher number of blocking pairs in the returned solution, as compared to the case when they are asked to generate solutions from scratch.

Table 7: Instability Rate (averaged across instances) in the (valid) matchings returned by LLMs when asked to resolve a given unstable matching of types One-BP and Random. The column “Baseline” contains the (average) Instability Rate for the provided matching of the indicated type. Numbers in bold indicate that the Instability Rate of the corrected solution is significantly worse than the provided matching. A * on the number in the One-BP column indicates a that Instability Rate is significantly lower than the case when a Random matching is provided (at $p < 0.05$).¹¹

Difficulty	Basic LLMs				Reasoning LLMs				Advanced Reasoning LLMs							
	Gemini-2.0-Flash		Llama-3.3-70B		Qwen-QwQ-32B		DeepSeek-70B		o3-mini		DeepSeek-R1		Gemini-2.5-Pro		Baseline	
	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random	One-BP	Random
Easy	45.5	46.3	34.25*	42.05	4.30*	10.80	8.74	9.05	0.25	0.00	0.20	0.20	0.30	0.80	10.00	77.32
Medium	55.09	58.81	57.48*	64.12	14.50*	49.20	23.39*	43.84	3.92	3.25	10.65	12.58	2.17	3.5	5.00	87.95
Hard	49.99*	74.66	61.42*	86.79	17.44*	86.91	35.38*	84.76	43.77*	59.98	14.8*	47.81	20.99*	25.08	2.00	94.52

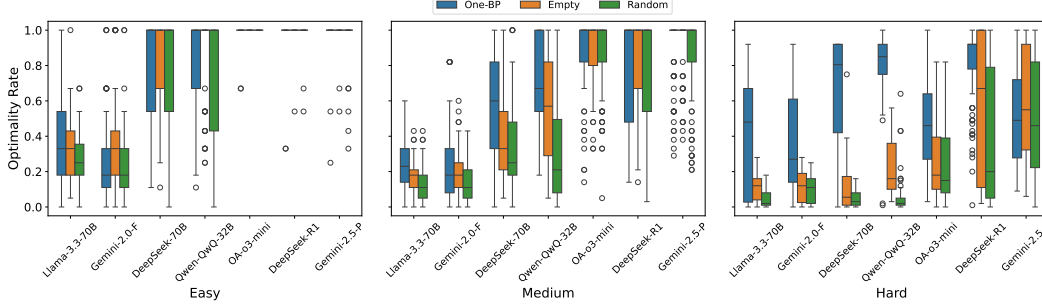


Figure 6: Optimality Rate when LLMs are asked to resolve a matching with a single blocking pair introduced into the Optimal solution (One-BP), or a randomly generated matching (Random). These are compared to the case when they are asked to generate a stable matching from scratch (Empty).

F Additional Material for Section 5

F.1 Comparing Impartial Culture and Master-List Instances

Generally, the classification of a preference profile as an impartial culture or master-list instance has a relatively small impact on the ability of an LLM to detect instability in the instance. However, we observe some differences between the ability of certain LLMs to detect stable/unstable matchings with ML instances compared to IC instances. Models such as DeepSeek-14B, o3-mini, and DeepSeek-70B are able to correctly detect stable solutions significantly more frequently with ML instances than with IC instances. A potential explanation for this is that Master-list preferences contain fewer *unique* preference lists, decreasing the chances that the model hallucinates blocking pairs. On the other hand, models such as Gemini-2.0-Flash and Llama-3.3-70B correctly identify unstable solutions significantly more often with IC preferences as compared to the case with ML preferences. The intuition for this observation is the opposite: with impartial culture preferences, there is a higher probability of having blocking pairs, therefore models that tend to predict that solutions are unstable will perform better with impartial culture instances.

G Fine-tuning Details

Models. We fine-tuned three reasoning models:

- DeepSeek-8B (deepseek-ai/DeepSeek-R1-Distill-Llama-8B),
- DeepSeek-14B (deepseek-ai/DeepSeek-R1-Distill-Qwen-14B), and
- Qwen-QwQ-32B (Qwen/QwQ-32B),

using the Unsloth¹³ framework with parameter-efficient tuning (LoRA).

Dataset. The dataset for the Generation task contains $N = 10000$ samples for which the reasoning trace is generated using a Python implementation of the DA algorithm. For the Preference Reasoning

¹³<https://unsloth.ai/>

task, the dataset consists of $N = 9000$ samples (3000 for each question level), where the reasoning trace involves explicitly identifying the positions of agents in the concerned preferences. In both datasets, we include an equal number of IC and ML instances, with sizes ranging from $n = 5$ to $n = 50$. Detailed examples of training examples for both tasks (Generation and Preference Reasoning) are provided in Appendix J.

Model Setup. We used the `FastLanguageModel.from_pretrained` interface from Unsloth to load the base model with a maximum sequence length of 10,000 tokens. The model was loaded in full precision (no quantization) and fine-tuned using Low-Rank Adaptation (LoRA) with the following settings:

- Rank (r): 32
- Target Modules: q_proj, k_proj, v_proj, o_proj, gate_proj, up_proj, down_proj
- LoRA α : 32
- LoRA Dropout: 0
- Bias: none
- Gradient Checkpointing: Enabled via `use_gradient_checkpointing="unsloth"`

Training Configuration. Fine-tuning was conducted using the `SFTTrainer` from the TRL library with the following training arguments:

- Epochs: 1
- Batch size per device: 2 (1, for Qwen-QwQ-32B)
- Gradient accumulation steps: 4 (2, for Qwen-QwQ-32B)
- Learning rate: 2×10^{-4} with a linear scheduler and 5 warmup steps
- Optimizer: AdamW-8bit
- Weight decay: 0.01
- Precision: Mixed precision (FP16 or BF16, based on hardware support)
- Seed: 3407

Hardware. Each model was fine-tuned using a single NVIDIA H100 GPU (80GB RAM) with CUDA support; model and inputs were explicitly transferred to GPU for inference and training.

Model Saving and Sharing. The resulting models were uploaded to the Hugging Face Hub and will be released upon acceptance.

Inference Setup. After fine-tuning, the model was evaluated using in-context inference. Inputs were formatted similarly to training prompts, and the model’s output was parsed to extract the JSON-formatted matching solution.

H Inference Details

Fine-tuned Models. The models we fine-tune have been pushed to HuggingFace Hub and will be released upon acceptance.

Inference Configuration. For the task of generating stable solutions, inference was performed on each of the open-source models such as DeepSeek-8B, DeepSeek-14B, Qwen-QwQ-32B, DeepSeek-70B, and Llama-3.3-70B, with the following sampling parameters:

- Temperature: 0.5
- Maximum tokens: 30,000
- Presence penalty: default
- Top-p: default

1176 Default values were utilized for all other sampling parameters. We used online APIs for the following
1177 models:

- 1178 • Gemini-2.0-Flash ('gemini-2.0-flash')
- 1179 • o3-mini ('o3-mini')
- 1180 • DeepSeek-R1 ('deepseek-reasoner')
- 1181 • Gemini-2.5-Pro ('gemini-2.5-pro-preview-03-25')

1182 **Hardware.** All inference experiments with open-source models were run on NVIDIA H100
1183 GPUs (80GB RAM) with CUDA support; model and inputs were explicitly transferred to GPU
1184 for inference and training. We used a single GPU for inference involving DeepSeek-8B and
1185 DeepSeek-14B, two GPUs for inference involving Qwen-QwQ-32B, and four GPUs for inference
1186 involving Llama-3.3-70B and DeepSeek-70B.

1187 I Prompts

1188 I.1 Example prompt for Generating Stable Solutions

1189 **Vanilla prompt.** The prompt used for generating stable solutions with LLMs follows standard
1190 prompting procedures, by first outlining the task, providing appropriate context, specifying constraints,
1191 and detailing the desired output format (a JSON object). Notice that despite the deferred-acceptance
1192 algorithm never being mentioned in the prompt, all models mentioned the deferred-acceptance
1193 algorithm in their responses.

You are an intelligent assistant who is an expert in algorithms. Consider the following instance of the two-sided matching problem, where 10 men are to be matched with 10 women. Here are the preference lists for all individuals:

```
<preferences>
{
  M: {
    M1: [W10,W1,W3,W6,W2,W4,W9,W8,W7,W5],
    M2: [W8,W3,W10,W6,W2,W5,W4,W7,W1,W9],
    ...
    M10: [W2,W5,W1,W3,W7,W6,W10,W4,W9,W8],
  },
  W: {
    W1: [M2,M8,M9,M10,M5,M7,M1,M4,M6,M3],
    W2: [M2,M7,M3,M1,M8,M9,M6,M10,M5,M4],
    ...
    W10: [M6,M4,M7,M5,M8,M9,M10,M2,M3,M1],
  }
}</preferences>
```

Your task is to find the proposer-optimal stable matching. You can use XML tags like `<scratchpad>` to explain your thought process while computing the solution. Once you have found a stable matching, please return your matching in the JSON format given below:

```
<answer>
{
  "M1": "<woman matched with M1>",
  "M2": "<woman matched with M2>",
  ...
  "M10": "<woman matched with M10>"
}</answer>
```

Make sure that each man/woman is matched with exactly ONE partner. It is mandatory that you provide a matching as a JSON object enclosed in `<answer></answer>` tags as described above.

1194

1195 **Providing Algorithmic Description.** The following is the prompt is a modification of the vanilla
 1196 prompt where the steps of the DA algorithm have been described to assist the model with implement-
 1197 ing the same.

You are an intelligent assistant who is an expert in algorithms.
 ...
 </preferences>
 Your task is to find the proposer-optimal stable matching. For this, you can use the Deferred Acceptance algorithm. The steps of this algorithm are described below:
 1. Initialize all men and women as unmatched.
 2. Create a list to keep track of each man's next proposal (initially set to 0 for all men).
 3. While there are unmatched men:
 a. Select an unmatched man (M).
 b. Find the next woman (W) on M's preference list that he hasn't proposed to yet.
 c. If W is unmatched, match M and W.
 d. If W is matched but prefers M to her current partner:
 - Unmatch W from her current partner.
 - Match M and W.
 - Set the unmatched man as W's previous partner.
 e. If W rejects M, move to the next woman on M's preference list.
 4. Repeat step 3 until all men are matched.
 You can use XML tags like <scratchpad> to explain your thought process ...
 ...
 It is mandatory that you provide a matching as a JSON object enclosed in
 <answer></answer> tags as described above.

1198

1199 **Modified Problem Setting.** The following is a modification to the vanilla prompt, where the setting
 1200 of *task-allocation* (assigning tasks to workers) is considered instead of the *stable-marriage* setting.
 1201 We replace *men* with *workers* and *women* with *tasks*.

You are an intelligent assistant who is an expert in algorithms. Consider the following instance
 of the two-sided matching problem, where 5 workers are to be assigned with 5 tasks, and
 each worker is assigned exactly one task.
 Here are the preference lists for all workers (W) over tasks (T) and the preferences of tasks
 over workers:
 <preferences>
 {
 W: {
 W1: [T5, T3, T4, T2, T1]
 ...
 }
 T: {
 T1: [W3, W5, W4, W1, W2]
 ...
 }
 }
 </preferences>
 Your task is to find a stable matching of workers and tasks. You can use XML tags like
 <scratchpad> to explain your thought process while computing the solution.
 Once you have found a stable matching, please return your matching in the JSON format
 given below:
 <answer>
 {
 "W1": "<task assigned to W1>",
 ...
 "W5": "<task assigned to W5>"
 }
 </answer>
 Make sure that each worker is assigned exactly ONE task. It is mandatory that you provide a
 matching as a JSON object enclosed in <answer></answer> tags as described above.

1202

1203 I.2 Example Prompts for Prompt Engineering

1204 I.2.1 CoT-Vanilla

1205 Chain-of-Thought methods were applied to the prompt in Appendix I.1 by additionally including an
1206 example trace of steps performed when running the deferred-acceptance algorithm on a randomly
1207 generated instance. The algorithm trace includes all proposals, all respective acceptances/rejections,
1208 and the resultant stable solution. The entire Chain-of-Thought example is enclosed within <example>
1209 XML tags.

You are an intelligent assistant who is an expert in algorithms. Your task is to find the proposer-optimal stable matching, for the two-sided matching problem. Here is an example to demonstrate how you should proceed:

<example>

<preferences>

```
{
M: {
M1: [W5,W1,W2,W4,W3],
M2: [W1,W2,W5,W4,W3],
M3: [W4,W2,W3,W1,W5],
M4: [W5,W1,W2,W4,W3],
M5: [W3,W5,W4,W2,W1],
},
W: {
W1: [M2,M3,M5,M4,M1],
W2: [M5,M2,M4,M3,M1],
W3: [M2,M1,M3,M5,M4],
W4: [M1,M4,M5,M3,M2],
W5: [M4,M3,M5,M2,M1],
}}
</preferences>
```

M1 is free. M1 proposes to W5

Since W5 is free, W5 accepts the proposal. Now M1 and W5 are matched.

M2 is free. M2 proposes to W1

Since W1 is free, W1 accepts the proposal. Now M2 and W1 are matched.

M3 is free. M3 proposes to W4

Since W4 is free, W4 accepts the proposal. Now M3 and W4 are matched.

M4 is free. M4 proposes to W5

Since W5 prefers M4 to their current partner M1, W5 accepts the proposal. Now M4 and W5 are matched, and M1 is free.

M1 is free. M1 proposes to W1

Since W1 prefers their current partner M2 to M1, W1 rejects the proposal. M2 and W1 are still matched, and M1 is still free.

M1 is free. M1 proposes to W2

Since W2 is free, W2 accepts the proposal. Now M1 and W2 are matched.

M5 is free. M5 proposes to W3

Since W3 is free, W3 accepts the proposal. Now M5 and W3 are matched.

<answer>

```
{
"M1": "W2",
"M2": "W1",
"M3": "W4",
"M4": "W5",
"M5": "W3"
}
```

</answer>

</example>

Consider the following instance of the two-sided matching problem, where 10 men are to be matched with 10 women ...

1210

1211 I.2.2 CoT-Shortlist

1212 The main distinction between the prompt described here and the one in Appendix I.2.1 lies in how the
 1213 algorithm's execution is detailed. In the CoT-Shortlist prompt, the provided algorithm trace includes
 1214 an additional step: agents remove each other from their respective shortlists if they become matched
 1215 with a partner they find more desirable than the other agents on their list. All other aspects of the
 1216 prompt are identical to the CoT-Vanilla prompt.

You are an intelligent assistant who is an expert in algorithms. Your task is to find the proposer-optimal stable matching, for the two-sided matching problem. Here is an example to demonstrate how you should proceed:

<example>

<preferences>

```
{
  M: {
    M1: [W4,W3,W5,W2,W1],
    M2: [W5,W4,W3,W1,W2],
    M3: [W5,W4,W1,W2,W3],
    M4: [W5,W4,W2,W1,W3],
    M5: [W2,W4,W5,W3,W1],
  },
  W: {
    W1: [M5,M2,M3,M4,M1],
    W2: [M3,M4,M5,M1,M2],
    W3: [M4,M1,M2,M5,M3],
    W4: [M5,M1,M4,M3,M2],
    W5: [M1,M4,M5,M3,M2],
  }
}
```

</preferences>

M1 is free. M1 proposes to W4. W4 accepts the proposal. Now M1 and W4 are matched. W1 deletes M4, M3, M2 from her list. M4, M3, M2 delete W4 from their list.

M2 is free. M2 proposes to W5. W5 accepts the proposal. Now M2 and W5 are matched.

M3 is free. M3 proposes to W5. W5 accepts the proposal. Now M3 and W5 are matched. W5 prefers M3, so W5 breaks her engagement with M2.

W3 deletes M2 from her list. M2 delete W5 from their list.

M4 is free. M4 proposes to W5. W5 accepts the proposal. Now M4 and W5 are matched.

W5 prefers M4, so W5 breaks her engagement with M3.

W4 deletes M5, M3 from her list. M5, M3 delete W5 from their list.

M5 is free. M5 proposes to W2. W2 accepts the proposal. Now M5 and W2 are matched.

W5 deletes M1, M2 from her list. M1, M2 delete W2 from their list.

<answer>

```
{
  "M1": "W4",
  "M2": "W3",
  "M3": "W1",
  "M4": "W5",
  "M5": "W2"
}
```

</answer>

</example>

Consider the following instance of the two-sided matching problem, where 10 men are to be matched with 10 women . . .

1217

1218 I.2.3 Few-shot Examples

1219 Few-shot prompting was applied to the prompt in Appendix I.1 by additionally including a series of
1220 randomly generated preference/stable solution pairs. As with other few-shot prompting strategies,
1221 the model is then asked to generate a stable solution (as shown in Appendix I.1). As with the CoT
1222 methods, each sample preference/stable solution pairs is enclosed in <example> XML tags.

You are an intelligent assistant who is an expert in algorithms. Your task is to find the proposer-optimal stable matching, for the two-sided matching problem. Here is an example to demonstrate how you should proceed:

<example>
<preferences>

```
{
M: {
M1: [W5,W3,W4,W2,W1],
M2: [W3,W4,W1,W2,W5],
M3: [W5,W1,W4,W2,W3],
M4: [W3,W2,W5,W1,W4],
M5: [W3,W4,W2,W1,W5],
},
W: {
W1: [M1,M4,M3,M5,M2],
W2: [M2,M4,M5,M1,M3],
W3: [M1,M2,M4,M5,M3],
W4: [M3,M5,M1,M4,M2],
W5: [M5,M3,M4,M2,M1],
}}
</preferences>
```

<answer>

```
{
"M1": "W3",
"M2": "W1",
"M3": "W5",
"M4": "W2",
"M5": "W4"
}
```

</answer>

</example>

<example>

...

</example>

<example>

...

</example>

Consider the following instance of the two-sided matching problem, where 10 men are to be matched with 10 women ...

1223

1224 I.3 Example Prompt for Evaluating Stability

1225 The following prompt requires LLMs to determine if a given solution to a provided preference profile
1226 is stable. Unlike the prompt in Appendix I.1, the only element that the LLM must include in their
1227 response is a binary response (yes/no).

Consider the following instance of the two-sided matching problem, where 5 men are to be matched with 5 women.

Here are the preference lists for all individuals:

<preferences>

```
{
  M: {
    M1: [W5,W3,W4,W2,W1],
    ...
  },
  W: {
    W1: [M3,M5,M4,M1,M2],
    ...
  }
}
```

</preferences>

Your task is to determine whether the following matching is stable or not.

<matching>

[[M1, W4],[M2, W5],[M3, W3],[M4, W1],[M5, W2],]

</matching>

Please return 'Yes' if you think the provided matching is stable and 'No' if you think it is unstable, and enclose your answer in <answer></answer> tags.

1228

1229 I.4 Example Prompts for Preference Comprehension

1230 In each of the following preference comprehension prompts, models are asked to provide the name
1231 of an agent (in level-1) or to provide a binary answer (yes/no for levels 2 and 3) in response to a
1232 provided question. In addition to changing the preference profiles for each instance of a preference
1233 comprehension task, the agents and positions mentioned in the question are also changed with each
1234 instance. For details about each level of preference comprehension, view Section 6.

1235 I.4.1 Level-1

Your goal is to correctly interpret the given preference lists and answer a specific question about agent preferences.

First, here are the preference lists for all individuals:

<preferences>

```
{
  M: {
    M1: [W5,W3,W4,W2,W1],
    ...
  },
  W: {
    W1: [M3,M5,M4,M1,M2],
    W2: [M1,M3,M4,M5,M2],
    ...
  }
}
```

</preferences>

Now, you will be asked a specific question about agent preferences:

<question>

Who is agent W2's, 1-most preferred agent?

</question>

Once you have determined the answer, provide your output in the following format:

1. The solution as a single agent name. For example, "W1"

Present your final answer within <answer> tags.

IMPORTANT: ONLY RETURN THE NAME OF THE SINGLE AGENT THAT IS THE ANSWER TO THE QUESTION. Do not include any explanations or additional information in your final answer.

1236

1237 **I.4.2 Level-2**

You are an AI assistant tasked with analyzing preference profiles in a two-sided matching problem with one-to-one solutions. Your goal is to correctly interpret the given preference lists and answer a specific question about agent preferences.

First, here are the preference lists for all individuals:

<preferences>

```
{
M: {
M1: [W5,W3,W4,W2,W1],
```

```
...
```

```
},
```

```
W: {
W1: [M3,M5,M4,M1,M2],
```

```
...
```

```
}}
```

</preferences>

Now, you will be asked a specific question about agent preferences:

<question>

Would agent W1, prefer M3 or M2 over M4?

</question>

Once you have determined the answer, provide your output in the following format:

1. The solution as a YES or a NO. For example, "NO"

Present your final answer within <answer> tags.

IMPORTANT: ONLY RETURN YES OR NO THAT IS THE ANSWER TO THE QUESTION. Do not include any explanations or additional information in your final answer.

1238

1239 **I.4.3 Level-3**

You are an AI assistant tasked with analyzing preference profiles in a two-sided matching problem with one-to-one solutions. Your goal is to correctly interpret the given preference lists and answer a specific question about agent preferences.

First, here are the preference lists for all individuals:

<preferences>

```
{
M: {
M1: [W5,W3,W4,W2,W1],
```

```
...
```

```
},
```

```
W: {
W1: [M3,M5,M4,M1,M2],
```

```
...
```

```
}}
```

</preferences>

Now, you will be asked a specific question about agent preferences:

<question>

If agent W1 is currently engaged to M4, would she accept proposals from M3 or M2?

</question>

Once you have determined the answer, provide your output in the following format:

1. The solution as a YES or a NO. For example, "NO"

Present your final answer within <answer> tags.

IMPORTANT: ONLY RETURN YES OR NO THAT IS THE ANSWER TO THE QUESTION. Do not include any explanations or additional information in your final answer.

1240

1241 **I.5 Example Prompt for Resolving Instability**

1242 For the task of resolving instability in a given unstable solution, the prompt begins by providing
1243 models with the instance's preference profile (as with the prompts for the other tasks). In addition,
1244 the prompt includes an unstable matching, and asks the model to resolve the instability by outputting
1245 a stable solution (in an identical format to the prompt in Appendix I.1).

You are an intelligent assistant who is an expert in algorithms. Consider the following instance of the two-sided matching problem and respective unstable matching, where 5 men are to be matched with 5 women.

Here are the preference lists for all individuals:

<preferences>

```
{
  M: {
    M1: [W5,W3,W4,W2,W1],
    M2: [W2,W3,W5,W1,W4],
    M3: [W5,W3,W1,W4,W2],
    M4: [W1,W3,W2,W5,W4],
    M5: [W2,W3,W4,W1,W5],
  },
  W: {
    W1: [M3,M5,M4,M1,M2],
    W2: [M1,M3,M4,M5,M2],
    W3: [M3,M2,M4,M1,M5],
    W4: [M4,M2,M3,M5,M1],
    W5: [M2,M4,M5,M1,M3],
  }
}
```

</preferences>

Here is an unstable matching.

<answer>

```
{
  "M1": "W4",
  "M2": "W5",
  "M3": "W3",
  "M4": "W2",
  "M5": "W1"
}
```

</answer>

Your task is to modify the given unstable matching to make it equivalent to the proposer-optimal stable matching. You can use XML tags like <scratchpad> to explain your thought process while computing the solution.

Once you have found a stable matching, please return your matching in the JSON format given below:

<answer>

```
{
  "M1": "<woman matched with M1>",
  "M2": "<woman matched with M2>",
  "M3": "<woman matched with M3>",
  "M4": "<woman matched with M4>",
  "M5": "<woman matched with M5>"
}
```

</answer>

Make sure that each man/woman is matched with exactly ONE partner. It is mandatory that you provide a matching as a JSON object enclosed in <answer></answer> tags as described above.

1246

1247 I.6 Example Prompt for Repeated Queries Due to Missing JSON Object

1248 For tasks where the desired output is a JSON object (when outputting a stable solution), models are
1249 given an additional opportunity to rectify issues in their response if the original response is incorrectly
1250 formatted. The prompt below is passed to the model to help rectify issues related to missing JSON
1251 objects. Note that the <initially passed prompt> and <last 3,000 characters of LLM's first response>
1252 XML tags are replaced by the initial prompt and the tail of the models initial response, respectively.

Previously, I gave you the following task:

<initially passed prompt>

In your response, you either failed to provide me with a matching or did not adhere to the JSON format I had asked for. Here are the last few lines of your response for reference:

<last 3,000 characters of LLM's first response>

Please correct your response and provide me with the matching in the following JSON format, enclosed in <answer></answer> tags.<answer>

```
{  
  "M1": "<woman matched with M1>",  
  "M2": "<woman matched with M2>",  
  "M3": "<woman matched with M3>",  
  "M4": "<woman matched with M4>",  
  "M5": "<woman matched with M5>"  
}
```

</answer>

Make sure that each man/woman is matched with exactly ONE partner.

1253

1254 I.7 Example Prompt for Repeated Queries Due to Incorrectly Formatted JSON Object

1255 Similar to the prompt in Appendix I.6, the following prompt is passed to the model when the model's
1256 initial response contains an incorrectly formatted JSON object. Once again, the <initially passed
1257 prompt> and <last 3,000 characters of LLM's first response> XML tags are replaced by the initial
1258 prompt and the tail of the models initial response, respectively.

Previously, I gave you the following task:

<initially passed prompt>

In your response, you failed adhere to the JSON format I had asked for. Here are the last few lines of your response for reference:

<last 3,000 characters of LLM's first response>

Please correct your response and provide me with the matching in the following JSON format, enclosed in <answer></answer> tags.<answer>

```
{  
  "M1": "<woman matched with M1>",  
  "M2": "<woman matched with M2>",  
  "M3": "<woman matched with M3>",  
  "M4": "<woman matched with M4>",  
  "M5": "<woman matched with M5>"  
}
```

</answer>

Make sure that each man/woman is matched with exactly ONE partner.

1259

1260 I.8 Example Prompt for Repeated Queries Due to Incomplete Matching

1261 Similar to the prompt in Appendix I.6, the following prompt is passed to the model when the model's
1262 initial response contains a correctly formatted JSON object, but the matching itself is incomplete, or
1263 some agents have multiple partners. After the initially passed prompt, note that additional details
1264 are provided to assist the LLM in rectifying its response. Again, the <initially passed prompt> and
1265 <last 3,000 characters of LLM's first response> XML tags are replaced by the initial prompt and the
1266 tail of the models initial response, respectively.

Previously, I gave you the following task:

<initially passed prompt>

In your response, the matching you selected involves some women being matched with multiple men, which is not allowed. For example, W2 is matched with M1, M2, and M5. Additionally, W3, and W4 are unmatched. Here are the last few lines of your response for reference:

<last 3,000 characters of LLM's first response>

Please correct your response and provide me with the matching in the following JSON format, enclosed in <answer></answer> tags.<answer>

```
{  
  "M1": "<woman matched with M1>",  
  "M2": "<woman matched with M2>",  
  "M3": "<woman matched with M3>",  
  "M4": "<woman matched with M4>",  
  "M5": "<woman matched with M5>"  
}
```

</answer>

Make sure that each man/woman is matched with exactly ONE partner.

1267

1268 J Training Examples for Fine-tuning

1269 J.1 System-prompt (s).

1270 This is the first part of the input, and is common across all tasks.

Below is an instruction that describes a task, paired with an input that provides further context. Write a response that appropriately completes the request.

Before answering, think carefully about the question and create a step-by-step chain of thoughts to ensure a logical and accurate response.

1271

1272 J.2 High-level instruction (u)

1273 • Generating:

Instruction:

You are an intelligent assistant who is an expert in algorithms. Your task is to find the proposer-optimal stable matching, for the two-sided matching problem.

Question:

Consider the following instance of the two-sided matching problem, where 5 men are to be matched with 5 women.

Here are the preference lists for all individuals:

1274

1275 • Comprehension:

Instruction:

You are an intelligent assistant who is an expert in algorithms. You will be given an instance of the two-sided matching problem, and will be asked to answer a question about the preferences of the agents involved.

Question:

First, here are the preference lists for all individuals:

1276

1277 **J.3 Preference Profile ($p^{(i)}$)**

```
<preferences>
{
  M: {
    M1: [W5,W3,W4,W2,W1],
    M2: [W2,W3,W5,W1,W4],
    M3: [W5,W3,W1,W4,W2],
    M4: [W1,W3,W2,W5,W4],
    M5: [W2,W3,W4,W1,W5],
  },
  W: {
    W1: [M3,M5,M4,M1,M2],
    W2: [M1,M3,M4,M5,M2],
    W3: [M3,M2,M4,M1,M5],
    W4: [M4,M2,M3,M5,M1],
    W5: [M2,M4,M5,M1,M3],
  }
}
</preferences>
</preferences>
```

1278

1279 **J.4 Task-prompt ($t^{(i)}$)**

1280 • **Generating:**

Your task is to find the proposer-optimal stable matching.

Once you have found a stable matching, please return your matching in the JSON format given below:

```
<answer>
{
  "M1": "<woman matched with M1>",
  "M2": "<woman matched with M2>",
  "M3": "<woman matched with M3>",
  "M4": "<woman matched with M4>",
  "M5": "<woman matched with M5>"
}
</answer>
```

Make sure that each man/woman is matched with exactly ONE partner. It is important that you enclose your JSON object in <answer></answer> tags.

1281

1282 • **Comprehension (Level-1):**

Now, you will be asked a specific question about agent preferences:
 <question>
 Who is agent W3's, 5-most preferred agent?
 </question>
 Once you have determined the answer, provide your output in the following format:
 1. The solution as a single agent name. For example, "W1"
 Present your final answer within <answer> tags.
 IMPORTANT: ONLY RETURN THE NAME OF THE SINGLE AGENT THAT IS THE
 ANSWER TO THE QUESTION. Do not include any explanations or additional infor-
 mation in your final answer.

1283

• **Comprehension (Level-2):**

Now, you will be asked a specific question about agent preferences:
 <question>
 Would agent W3, prefer M4 and M3 over M1?
 </question>
 Once you have determined the answer, provide your output in the following format:
 1. The solution as a YES or a NO. For example, "NO"
 Present your final answer within <answer> tags.
 IMPORTANT: ONLY RETURN YES OR NO THAT IS THE ANSWER TO THE QUES-
 TION. Do not include any explanations or additional information in your final answer.

1285

• **Comprehension (Level-3):**

Now, you will be asked a specific question about agent preferences:
 <question>
 If agent W5 is currently engaged to M3, would she accept proposals from M1 and
 M4?
 </question>
 Once you have determined the answer, provide your output in the following format:
 1. The solution as a YES or a NO. For example, "NO"
 Present your final answer within <answer> tags.
 IMPORTANT: ONLY RETURN YES OR NO THAT IS THE ANSWER TO THE QUES-
 TION. Do not include any explanations or additional information in your final answer.

1287

1288 The input $x^{(i)}$ ends with the following text, to indicate the start of the chain-of-thought reasoning
 1289 trace,

Response:
 <think>

1290

1291 **J.5 Reasoning-trace ($r^{(i)}$):**

• **Generating:**

1292

Response:

<think>

Okay, so I need to find a stable matching for 5 men and 5 women. For this, I can use the Gale-Shapley algorithm. Before I compute the solution for this instance, I will list the steps of the Gale-Shapley algorithm.

1. Initialize all men and women as unmatched.
2. Create a list to keep track of each man's next proposal (initially set to 0 for all men).
3. While there are unmatched men:
 - a. Select an unmatched man (M).
 - b. Find the next woman (W) on M's preference list that he hasn't proposed to yet.
 - c. If W is unmatched, match M and W.
 - d. If W is matched but prefers M to her current partner:
 - Unmatch W from her current partner.
 - Match M and W.
 - Set the unmatched man as W's previous partner.
 - e. If W rejects M, move to the next woman on M's preference list.
4. Repeat step 3 until all men are matched.

Next, I will execute the above algorithm for the given instance. The steps are as follows:

M1 is free. M1 proposes to W3

Since W3 is free, W3 accepts the proposal. Now M1 and W3 are matched.

M2 is free. M2 proposes to W5

Since W5 is free, W5 accepts the proposal. Now M2 and W5 are matched.

M3 is free. M3 proposes to W1

Since W1 is free, W1 accepts the proposal. Now M3 and W1 are matched.

M4 is free. M4 proposes to W4

Since W4 is free, W4 accepts the proposal. Now M4 and W4 are matched.

M5 is free. M5 proposes to W4

Since W4 prefers their current partner M4 to M5, W4 rejects the proposal. M4 and W4 are still matched, and M5 is still free.

M5 is free. M5 proposes to W1

Since W1 prefers M5 to their current partner M3, W1 accepts the proposal. Now M5 and W1 are matched, and M3 is free.

M3 is free. M3 proposes to W4

Since W4 prefers their current partner M4 to M3, W4 rejects the proposal. M4 and W4 are still matched, and M3 is still free.

M3 is free. M3 proposes to W3

Since W3 prefers their current partner M1 to M3, W3 rejects the proposal. M1 and W3 are still matched, and M3 is still free.

M3 is free. M3 proposes to W2

Since W2 is free, W2 accepts the proposal. Now M3 and W2 are matched.

</think>

1293

1294

• **Comprehension (Level-1):**

Alright, I have been given the task of determining W3's 5-most preferred agent. For this, let me look at her preference list in order:

position 1: M4

position 2: M5

position 3: M1

position 4: M3

position 5: M2

Hence, W3's 5-most preferred agent is M2.

</think>

1295

1296

• **Comprehension (Level-2):**

Alright, I have been asked to determine whether agent W3 would prefer M4 and M3 over M1. For this, let me look at W3's preference list in order:
 position 1: M4
 position 2: M5
 position 3: M1
 position 4: M3
 position 5: M2
 So, M4 is W3's 1-most preferred partner, M3 is W3's 4-most preferred partner, and M1 is W3's 3-most preferred partner.
 Hence, agent W3 prefers M4 over M1, and agent W3 does not prefer M3 over M1.
 This means that the order of preference is $M4 > M1 > M3$.
 Therefore, since the question was whether agent W3 would prefer BOTH M4 AND M3, given that it DOES NOT prefer M3 over M1, the answer is 'NO'.
 </think>

1297

1298

• **Comprehension (Level-3):**

Alright, I have been asked to determine whether agent W5 would accept a proposal from M1 or M4, if she is currently matched with M3. For this, let me look at W5's preference list in order:
 position 1: M5
 position 2: M4
 position 3: M2
 position 4: M3
 position 5: M1
 So, M1 is W5's 5-most preferred partner, M4 is W5's 2-most preferred partner, and M3 is W5's 4-most preferred partner.
 Hence, agent W5 does not prefer M1 over their current partner M3, and agent W5 prefers M4 over their current partner M3. This means that the order of preference is $M4 > M3 > M1$.
 Therefore, since the question was whether agent W5 would prefer EITHER M1 OR M4, given that it prefers M4 over M3, the answer is 'YES'.
 </think>

1299

1300 **J.6 Answer ($a^{(i)}$):**

1301

• **Generating:**

```
<answer>
{
  "M1": "W3",
  ...
  "M5": "W1"
}
</answer>
```

1302

1303

• **Comprehension (Level-1):**

```
<answer>M2</answer>
```

1304

1305

• **Comprehension (Level-2):**

```
<answer>NO</answer>
```

1306

1307

• **Comprehension (Level-3):**

```
<answer>YES</answer>
```

1308