Acknowledgements

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Checklist

- 1. For all authors...
 - (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
 - (b) Did you describe the limitations of your work? [Yes] See Appendix E.
 - (c) Did you discuss any potential negative societal impacts of your work? [Yes] We include additional details about broader impact in the supplementary material.
 - (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
- 2. If you are including theoretical results...
 - (a) Did you state the full set of assumptions of all theoretical results? [Yes]
 - (b) Did you include complete proofs of all theoretical results? [Yes]
- 3. If you ran experiments...
 - (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] We provide link to our code to replicate all experiments
 - (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See sections B and A in the supp. material.
 - (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] We include error estimates in all figures.
 - (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] We estimate compute in section A in the supp. material.
- 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
 - (a) If your work uses existing assets, did you cite the creators? [Yes]
 - (b) Did you mention the license of the assets? [Yes]
 - (c) Did you include any new assets either in the supplemental material or as a URL? [N/A]
 - (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
 - (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
- 5. If you used crowdsourcing or conducted research with human subjects...
 - (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
 - (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
 - (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

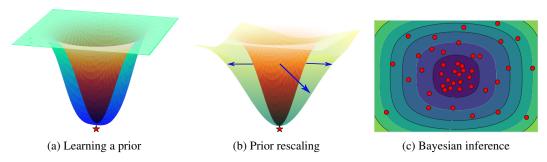


Figure 5: **Bayesian transfer learning pipeline.** We (a) learn a prior by fitting a probability distribution over feature extractor parameters to a pre-training posterior mode, (b) rescale the prior for a downstream task, and (c) use the prior for Bayesian inference on downstream tasks.

A Computational Considerations

While our method requires a learned prior and a re-scaling coefficient, it actually adds very little additional cost on top of standard fine-tuning routines. We now evaluate the costs of each of the three stages of our pipeline: (i) inferring the posterior on the source task; (ii) re-scaling the posterior to become an informative prior for the downstream task; (iii) using the informative prior in the downstream task.

For (i), we note that we used SWAG because it has already been well-established as a tractable, well-studied, and simple way to infer a non-diagonal posterior, and it can be used starting from a pre-trained checkpoint [34]. We apply SWAG to a pre-trained checkpoint. We then do 50 iterations of burn-in and 10 cycles of 200 iterations each for a total of 2050 iterations with a batch size of 128. In total, inferring the posterior costs only around a quarter of an epoch of training on ImageNet, and the computations in this step can be entirely avoided by users releasing pre-trained SWAG posteriors, as we do for all of the tasks in our paper.

For (ii), we only need to tune a single re-scaling hyperparameter. When we perform MAP optimization with our informative priors, tuning this scalar only adds $\frac{1}{7}$ of the total runtime of transfer learning (including tuning other hyperparameters such as learning rate). For example, full hyperparameter tuning (including the re-scaling coefficient) and running MAP on the full Oxford Flowers-102 dataset takes 20 hours in total, whereas hyperparameter tuning and running traditional SGD-based transfer learning takes 17 hours in total on a machine with a single NVIDIA A100 GPU, which is a comparable amount of hyperparameter tuning to other works and significantly less than others [32, 6]. We also note that our tuning is done with a simple validation grid-search, exactly the same way as any other hyperparameter tuning in standard transfer learning, and does not require any special expertise.

For (iii), we offer two options, (a) MAP optimization of the downstream posterior using the informative prior, and (b) full Bayesian inference with SGLD or SGHMC using the downstream posterior. (a) costs the same as standard transfer learning both at train and test time, and we show it works better than standard transfer learning. (b) Bayesian inference with our informative priors bears the same cost as deep ensembles or as Bayesian inference with non-learning priors, which we compare to in our work. A single chain of SGLD incurs the same cost as a single run of SGD, since the gradient of the log prior density can be computed without automatic differentiation in the same fashion as weight-decay, which we are replacing. We also compare multiple chains of SGLD with deep ensembles, which require the same cost, and show that Bayesian inference with our learned prior works significantly better.

B Classification

B.1 Implementation Details

Based on the evaluation protocols in the papers introducing the datasets, we report the top-one accuracy for CIFAR-10 and CIFAR-100 and mean per-class accuracy for Oxford-IIIT Pets and Oxford 102 Flowers. For Oxford 102 Flowers, we select hyperparameters based on the validation

sets specified by the dataset creators. While tuning hyperparameters, we hold out a subset of the training set for validation on the other datasets. After selecting the optimal hyperparameters from the validation set, we retrain the model using the selected parameters on both the training and validation images. Test results are reported.

We use ResNet-50 and ResNet-101 architectures [18] with supervised and SSL checkpoints [6, 48] pre-trained on the ImageNet 1k dataset [9]. We use the same hyperparameters as in Chen et al. [6].

Learning the prior. In order to learn the prior, we use the SWAG algorithm [34] with cyclic learning rate presented in Zhang et al. [53]. We use SGD with a Nesterov momentum parameter of 0.9 for optimization. We select our initial learning rate from a logarithmic grid between 0.005 and 0.5 and evaluate 10000 iterations for each cycle. Other hyperparameters are from Chen et al. [6], including data augmentation which comprises color augmentation, blurring, random crops, and horizontal flips.

Downstream classification tasks. We train for 30, 000 steps with a batch size of 128 on CIFAR-10 and CIFAR-100, 16 for Oxford Flowers-102, and 32 for Oxford-IIIT Pets. We use SGD and SGHMC with momentum parameter of 0.9. During fine-tuning, we perform random crops with resizing to 224×224 and horizontal flips. At test time, we resize the images to 256 pixels along the shorter side and produce a 224×224 center crop. In our study, we select the learning rate and weight decay from a grid of 7 logarithmically spaced learning rates between 0.0001 and 0.1, and 7 logarithmically spaced weight decay values between $1e^{-6}$ and $1e^{-2}$, as well as without weight decay. These weight decay values are divided by the learning rate. For the SGHMC optimizer, we select the temperature from a logarithmic grid of 8 values between $1e^{0}$ and $1e^{-8}$, and we use predictions from 5 different chains for the final evaluation.

B.2 Supervised Pre-Training

As mentioned above, we conduct additional experiments with priors learned on labeled data. Figures 7 and 6 contain downstream task performance comparisons for Resnet-50 and Resnet-101 models with priors learned starting with torchvision checkpoints pre-trained on ImageNet 1k. These figures compare our method to SGD transfer learning with pre-trained initializations and Bayesian inference with non-learned Gaussian prior. Across both backbones, our method outperforms all baselines for nearly every number of train samples and for all four datasets, where the Bayesian inference model with a non-learned prior is always worse. (For full results see tables 2, 3, 4 and 5.

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	75.7	46.8	24.9	9.2	4.3
BNN Non-learned Prior	86.5	68.1	44.9	11.6	4.8
SGD Ensemble	78.7	57.2	31.6	10.9	5.2
SGD Transfer Init	81.9	62.1	36.5	11.6	4.4

Table 2: CIFAR-10 test error with torchvision Resnet-50 prior/initialization.

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	86.8	64.9	38.8	26.1	17.2
BNN Non-learned Prior	97.0	83.6	50.8	30.1	19.2
SGD Ensemble	93.3	74.6	43.1	26.4	17.4
SGD Transfer Init	94.9	71.8	42.7	26.9	17.8

Table 3: CIFAR-100 test error with torchvision Resnet-50 prior/initialization.

B.3 Self-Supervised Pre-Training

In this section, we present additional data from Figures 3 and 4 found in the main body. Tables 10, 11, 12 and 13 correspond to Figure 3, while Table 14 corresponds to Figure 4.

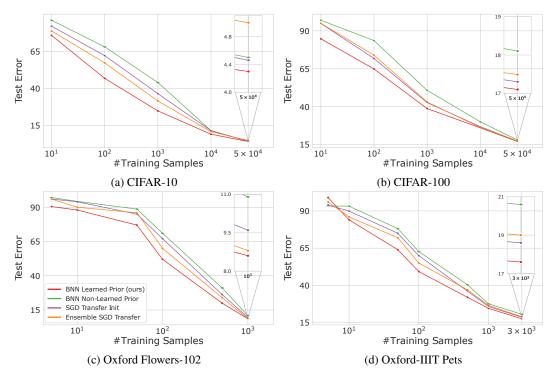


Figure 6: **Performance comparison.** The test error of a ResNet-50 BNN equipped with supervised pre-trained prior (red) is consistently lower than that of standard SGD-based transfer learning from the same pre-trained checkpoint (purple), an equivalently sized ensemble of SGD-based transfer learning models (orange), or BNN with a mean zero isotropic Gaussian prior (green). The x-axis denotes the number of downstream training samples. Axes on a logarithmic scale.

Model/# samples	5	10	50	100	500	1000
BNN Learned Prior	90.5	88.0	77.2	52.8	20.4	8.5
BNN Non-learned Prior	97.1	94.3	88.7	71.0	35.0	10.7
SGD Ensemble	96.2	90.1	86.1	60.0	24.0	8.9
SGD Transfer Init	96.6	94.3	85.4	68.0	26.3	9.6

Table 4: Oxford-Flowers-102 test error with torchvision Resnet-50 prior/initialization.

B.4 Evaluating uncertainty

In Figure 8, we present the test negative log likelihood (NLL) for both the Bayesian methods and SGD-based methods for 4 datasets (CIFAR-10, CIFAR-100, Oxford Flowers-102 and Oxford-IIIT Pets). In general, we can see a similar trend for all the datasets where Bayesian transfer learning outperforms all other methods. In Figure 9, we present the reliability diagrams for CIFAR-10 and CIFAR-100 for ResNet-18. A perfectly calibrated network would express no difference between accuracy and confidence, represented by a dashed black line. Under-confident predictions are those below this line, whereas overconfident predictions are those above. We can see that Bayesian inference with learned priors are the best calibrated among the methods we consider. The error bars are computed on 5 runs, and the radius is one standard error.

C Semantic Segmentation

C.1 Implementation Details

For our semantic segmentation evaluations, we use the DeepLabv3+ [4] model with ResNet-50 and ResNet-101 backbone architectures. We train our models using Pascal VOC 2012 and Cityscapes trainsets and evaluate on their *val* sets. We utilize the same hyperparameters employed in Chen et al.

	Model/# samples	5	10	50	100	500	1000	3000	
	BNN Learned Prior	98.2	84.0	64.7	49.3	32.0	24.6	17.6	
	BNN Non-learned Prior	93.4	93.2	78.1	62.7	45.4	38.6	25.2	
	SGD Ensemble	96.6	86.3	72.1	55.0	37.9	26.6	19.3	
	SGD Transfer Init	94.5	89.9	75.4	61.1	36.1	25.9	18.6	
Ta	Table 5: Oxford-IIIT Pets test error with torchvision Resnet-50 prior/initialization.								

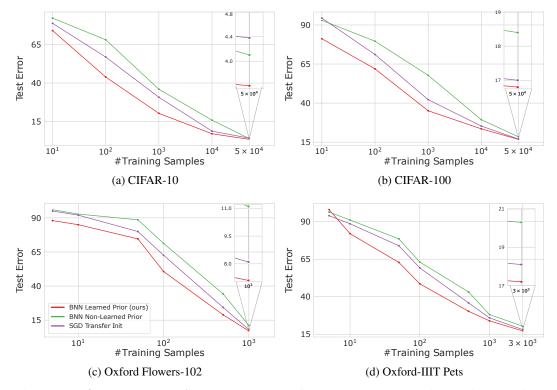


Figure 7: **Performance comparison.** The test error of a ResNet-101 BNN equipped with supervised pre-trained prior (red) is consistently lower than that of standard SGD-based transfer learning from the same pre-trained checkpoint (purple), or BNN with a mean zero isotropic Gaussian prior (green). The x-axis denotes the number of downstream training samples. Axes on a logarithmic scale.

[4] with a few modifications: our learning rate schedule is the *poly* policy [3] with initial value 0.01 for Pascal VOC 2012 and 0.1 for Cityscapes, our crop size is 513×513 for Pascal VOC 2012 and 768×768 for Cityscapes. Our *output stride* is 16 for both training and evaluating, and we perform random scale data augmentation during training. In our SGD and SGLD optimization, we employ Nesterov momentum, and we set the momentum coefficient to 0.9. We use batches of 16 images and weight decay with a coefficient of 10^{-4} . In addition, for the SGLD optimizer, the temperature is set to 2×10^{-8} . We train all models for 30k iterations.

C.2 MAP Estimation

We also compare MAP estimates (SGD) on the loss induced by our learned priors to the baseline with only weight decay. Table 15 and 16 contain the results using ResNet-50 and ResNet-101 backbones, respectively. On both architectures and both datasets, learned priors boost performance on these experiments without using Bayesian inference at all.

D Alternative Methods for Re-Scaling the Prior

As discussed previously, we tune a coefficient to re-scale our learned prior by maximizing the validation log-likelihood over a grid of coefficient values. Our objective in this section is to examine

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	73.9	43.9	20.4	7.2	3.7
BNN Non-learned Prior	82.1	68.7	36.3	16.4	4.1
SGD Transfer Init	78.6	56.8	30.8	8.9	4.3
(OTELD 10			D	101 '	1 1.

Table 6: CIFAR-10 test error with torchvision Resnet-101 prior/initialization.

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	81.2	61.9	35.1	23.5	16.8
BNN Non-learned Prior	93.5	79.6	58.8	32.0	20.2
SGD Transfer Init	94.4	71.1	44.2	25.9	17.0
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Table 7: CIFAR-100 test error with torchvision Resnet-101 prior/initialization.

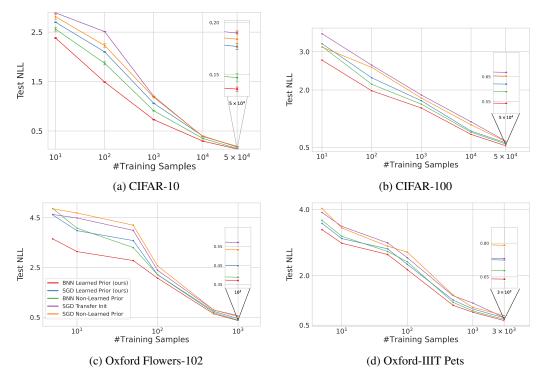


Figure 8: **Predictive likelihood.** The horizontal axis denotes the number of downstream CIFAR-10 training samples on a logarithmic scale. All experiments performed with ResNet-50 backbone.

three alternative methods for tuning coefficients to re-scale the prior. First, we consider maximizing the Laplace approximation to the marginal likelihood, which does not require holding out validation data [22]. Marginal likelihood maximization can be performed online together with MAP estimation in deep learning. Specifically, we try tuning a single coefficient and also per-layer coefficients for re-scaling the prior's covariance matrix in this fashion. A third alternative is to instead tune two coefficients: one for the diagonal component of the covariance matrix, and one for the low-rank component. We use a grid-search in this case to tune the two coefficients. Table 17 contains the results for these three different methods as well as the method advocated in the main body of our draft using the SimCLR Resnet-50 backbone and SGHMC for downstream fine-tuning. We observe that on most datasets, tuning a single covariance re-scaling hyperparameter using marginal likelihood does not improve the results reported in the paper. Using different coefficients for each layer results in slight improvements on some datasets. On CIFAR-10, Oxford Flowers-102, and Oxford-IIIT Pets, test errors decreased by 0.03% - 0.08%. Finally, on most datasets, re-scaling the diagonal and low-rank components of the covariance matrix independently results in slightly better performance. For example, the test errors on CIFAR-100, Oxford Flowers-102, and Oxford-IIIT Pets decrease

Model/# samples	5	10	50	100	500	1000
BNN Learned Prior	88.0	81.0	74.6	50.7	18.9	7.1
BNN Non-learned Prior	96.0	92.8	88.6	75.1	41.1	21.7
SGD Transfer Init	95.0	90.2	80.8	59.2	22.3	8.6
0 0 1 11 100	• •				101	

Table 8: Oxford-Flowers-102 test error with torchvision Resnet-101 prior/initialization.

5	10	50	100	500	1000	3000
96.1	82.4	62.9	48.6	30.3	23.9	17.2
96.4	91.1	78.6	67.7	56.9	38.2	26.2
94.0	88.6	74.3	59.2	35.5	25.0	18.1
	96.4	96.4 91.1	96.4 91.1 78.6	96.1 82.4 62.9 48.6 96.4 91.1 78.6 67.7	96.1 82.4 62.9 48.6 30.3 96.4 91.1 78.6 67.7 56.9	96.1 82.4 62.9 48.6 30.3 23.9 96.4 91.1 78.6 67.7 56.9 38.2

Table 9: Oxford-IIIT Pets test error with torchvision Resnet-101 prior/initialization.

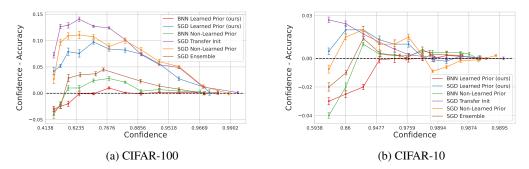


Figure 9: **Reliability diagrams** - Bayesian transfer learning is capable of significantly improving calibration over standard transfer training (SGD Transfer Init) as well as non-learned prior BNN. Error bars are computed over 5 runs.

by 0.03% - 0.11%. We note that these alternative methods bear an additional computational and implementational cost.

E Limitations

Limitations. Bayesian model averaging provides particularly compelling results, especially in terms of calibration and data efficiency, but does incur some additional computational cost. On the other hand, our SGLD based sampling procedures outperform deep ensembles, which have comparable computational costs. Moreover, our informative priors with MAP optimization still provide a clear performance boost over standard transfer learning, with negligible overhead. In terms of applications, we have limited our considerations to vision settings, for a focused exposition.

Directions for future research. Pre-training has been particularly transformative in natural language processing, where informative priors could be used to great effect. More broadly studying loss-surface alignment between tasks could also be informative for understanding how to build models that provide good domain generalization and robustness to spurious correlations.

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	73.8 ± 3.2	43.1 ± 3.1	21.6 ± 1.6	7.8 ± 0.5	4.0 ± 0.1
SGD Learned Prior	75.9 ± 2.1	49.3 ± 1.9	26.8 ± 1.1	8.9 ± 0.3	4.2 ± 0.1
BNN LP Single-Chain	74.0 ± 2.7	48.4 ± 2.8	25.4 ± 1.3	8.3 ± 0.5	4.2 ± 0.2
BNN LP Laplace	86.9 ± 3.1	66.4 ± 2.2	37.1 ± 2.1	11.2 ± 0.6	4.6 ± 0.2
BNN Non-learned Prior	86.1 ± 3.9	68.0 ± 3.8	44.9 ± 1.7	11.6 ± 0.6	4.4 ± 0.1
SGD Ensemble	81.3 ± 2.2	57.0 ± 3.1	32.3 ± 1.4	9.4 ± 0.5	4.6 ± 0.1
SGD Transfer Init	83.6 ± 1.8	61.0 ± 2.1	36.8 ± 1.0	10.9 ± 0.4	4.4 ± 0.1
Table 10: CIEAD 10	tost arror wit	h SimCI D (S	SL) Deepet 50	nrior/initializ	ation

Table 10: CIFAR-10 test error with SimCLR (SSL) Resnet-50 prior/initialization.

Model/# samples	10	100	1000	10000	50000
BNN Learned Prior	81.9 ± 3.7	63.8 ± 2.4	37.3 ± 1.2	24.2 ± 0.7	16.4 ± 0.4
SGD Learned Prior	90.9 ± 2.8	69.7 ± 1.9	41.1 ± 1.1	26.3 ± 0.5	17.3 ± 0.4
BNN LP Laplace	90.1 ± 3.1	73.6 ± 2.5	42.8 ± 1.9	27.9 ± 1.2	18.1 ± 0.8
BNN LP Single-Chain	84.4 ± 3.8	69.6 ± 2.1	$38.2 \pm 2.$	25.1 ± 0.7	16.8 ± 0.5
BNN Non-learned Prior	97.0 ± 3.9	84.0 ± 2.8	50.8 ± 1.8	28.9 ± 0.8	18.8 ± 0.6
SGD Ensemble	89.2 ± 2.0	70.9 ± 1.7	44.0 ± 1.2	26.6 ± 0.7	17.2 ± 0.4
SGD Transfer Init	93.0 ± 2.2	73.2 ± 1.8	45.5 ± 0.9	27.8 ± 0.6	17.9 ± 0.3
Table 11: CIEAD 10	1 tost arror w	th SimCI D (SCI) Deepet 5	0 prior/initial	rotion

Table 11: CIFAR-100 test error with SimCLR (SSL) Resnet-50 prior/initialization.

Model/# samples	5	10	50	100	500	1000
BNN Learned Prior	89.8 ± 3.1	80.3 ± 1.7	67.3 ± 1.8	48.9 ± 1.4	19.7 ± 0.7	7.8 ± 0.2
SGD Learned Prior	90.2 ± 2.0	81.0 ± 1.3	67.0 ± 1.1	49.9 ± 1.1	20.6 ± 0.6	8.2 ± 0.1
BNN LP Laplace	97.4 ± 2.3	93.1 ± 2.1	89.9 ± 1.4	69.8 ± 1.4	32.1 ± 0.8	11.9 ± 0.2
BNN LP Single-Chain	90.3 ± 2.8	80.8 ± 1.8	67.4 ± 1.1	49.5 ± 1.7	20.1 ± 0.9	8.1 ± 0.2
BNN Non-learned Prior	97.1 ± 2.7	94.3 ± 2.0	88.7 ± 2.1	71.0 ± 2.0	35.0 ± 0.9	10.9 ± 0.3
SGD Ensemble	93.8 ± 2.0	91.2 ± 1.6	84.3 ± 1.2	56.8 ± 1.0	21.9 ± 0.3	8.5 ± 0.2
SGD Transfer Init	96.0 ± 1.8	93.0 ± 1.8	82.0 ± 1.3	66.0 ± 1.1	24.1 ± 0.6	8.7 ± 0.1
	100	1.1 01 0	DID (COL) D			

Table 12: Oxford-Flowers-102 test error with SimCLR (SSL) Resnet-50 prior/initialization.

Model/# samples	5	10	50	100	500	1000	3000
BNN Learned Prior	89.4 ± 2.6	80.9 ± 1.9	62.0 ± 1.8	47.3 ± 1.6	30.7 ± 1.1	21.4 ± 0.9	17.1 ± 0.7
SGD Learned Prior	91.7 ± 1.9	84.7 ± 1.7	66.1 ± 1.9	51.2 ± 1.3	34.1 ± 0.8	23.9 ± 0.6	18.2 ± 0.6
BNN LP Laplace	94.1 ± 3.2	91.1 ± 2.7	75.9 ± 2.7	60.8 ± 1.7	40.1 ± 1.4	26.8 ± 1.4	21.3 ± 0.3
BNN LP Single-Chain	91.0 ± 2.7	84.9 ± 2.5	65.2 ± 2.9	49.4 ± 1.9	31.9 ± 1.8	23.7 ± 1.6	18.1 ± 0.6
BNN Non-learned Prior	93.4 ± 2.9	93.2 ± 2.1	78.1 ± 2.2	62.7 ± 2.1	45.4 ± 1.1	38.6 ± 1.1	25.2 ± 0.8
SGD Ensemble	92.2 ± 2.1	88.1 ± 1.4	68.3 ± 2.1	55.7 ± 1.5	35.0 ± 1.2	24.5 ± 1.0	18.1 ± 0.5
SGD Transfer Init	94.9 ± 2.1	88.5 ± 1.6	72.0 ± 1.3	55.7 ± 1.1	36.3 ± 1.0	25.0 ± 0.7	18.9 ± 0.3
Table 13: Oxford	IIIT Date tae	terror with	SimCL P (SS	I) Recnet 5	0 prior/initia	lization	

Table 13: Oxford-IIIT Pets test error with SimCLR (SSL) Resnet-50 prior/initialization.

$2.0 59.9 \pm 1.5$	$.7 43.0 \pm 1.3$	107100	
	$.1 + 3.0 \pm 1.0$	18.7 ± 0.8	8.8 ± 0.4
$1.4 63.9 \pm 1.4$	$.5 \mid 48.0 \pm 1.2$	20.1 ± 0.8	9.8 ± 0.3
2.3 78.5 ± 1.5	.8 59.0 \pm 1.3	32.0 ± 1.0	11.3 ± 0.4
1.6 69.0 ± 1.3	$.3 52.0 \pm 1.1$	21.3 ± 0.6	10.3 ± 0.2
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Table 14: Classification error with self-supervised pre-learning - CIFAR-10.1

Table 15: **MAP estimation for semantic segmentation (ResNet-50 backbone).** Comparing the performance (Mean-IoU) of SGD with learned priors (both torchvision supervised and SimCLR SSL) to a non-learned prior (weight decay with torchvision initialization) over DeepLabv3+ backbone parameters for downstream segmentation with SGD rather than Bayesian inference. Evaluations are conducted on *val* sets.

Dataset	Non-Learned Prior MAP	Supervised Prior MAP	SSL Prior MAP
PASCAL VOC 2012	73.17	73.27	73.48
Cityscapes	75.52	75.57	76.15

Table 16: **MAP estimation for semantic segmentation (ResNet-101 backbone).** Comparing the performance (Mean-IoU) of SGD with torchvision learned priors to a non-learned prior (weight decay with torchvision initialization) over DeepLabv3+ backbone parameters for downstream segmentation with SGD rather than Bayesian inference. Evaluations are conducted on *val* sets.

Dataset	Non-Learned Prior MAP	Supervised Prior MAP
PASCAL VOC 2012	75.53	75.89
Cityscapes	77.04	77.27

Table 17: Alternative methods for prior re-scaling (SimCLR ResNet-50 backbone) - test error corresponding to different methods for re-scaling the prior.

Dataset	CIFAR-100	CIFAR-10	Oxford Flowers-102	Oxford-IIIT Pets	
Single Coefficient - Grid Search	17.14	4.02	7.85	17.12	
Single Coefficient - Marginal Likelihood	17.13	4.02	7.87	17.06	
Per-Layer - Marginal Likelihood	17.14	3.99	7.79	17.05	
Separate Scales for Low-Rank and Diagonal	17.11	4.01	7.81	17.03	