

1 **To all Reviewers** Thanks you for the insightful reviews. Reviewers 2 and 3 question the novelty and claim that
2 we merely apply known results. Prior to our work, there were *no* first-order method (FOM) for computing market
3 equilibria that converge linearly. Previous methods achieving (partial) linear convergence either (i) involve layers of
4 oracle abstraction (with costly oracle calls) (Bei et al. - Ascending-Price Algorithms...) or (ii) only exhibits initial linear
5 convergence under restrictive “large-market” assumptions (Cole & Tao - Balancing the Robustness...). Thus, the fact
6 that we are able to achieve a linear rate with something as simple as proximal (projected) gradient (PG) is, in our view, a
7 *strength* rather than weakness. Linear convergence of PG is not obvious either: applying the standard theory naively to
8 Eisenberg-Gale only yields a $1/T$ rate, and even this already requires our Lemma 1 through 3 in order to get a Lipschitz
9 constant. To get our linear rate (Thm 5), it is necessary to first realize that strong convexity and Lipschitz gradients can
10 be proved in the EG utility space using properties of market equilibrium (nobody seems to have realized this crucial
11 property before; a priori one only gets strict convexity), and utilizing our new bounds on equilibrium utilities (Lemma
12 1). This makes the reformulated problem satisfy the PL inequality (Eq. (13)). Finally, minor adjustments to existing
13 theorems are needed to tackle our setting. Similar ideas are needed in the case of QL and Leontief utilities. For all
14 cases, our new market-specific bounds on the equilibrium quantities (Lemma 1-3) are crucial as they ensure explicit
15 Lipschitz constants on gradients. In our paper, we will also make sure to clarify that these steps are indeed necessary.
16 Note also that for Thm 2 & 3, it was necessary to refine the known convergence guarantees, as explained in the texts (l.
17 590 & 631 in their proofs). Finally, we propose a practical linesearch scheme (Algorithm 1) which we implement in
18 the experiments. We give convergence guarantees (Theorem 4 & 10) under the Proximal-PL condition beyond strong
19 convexity. These are entirely new.

20 **To Reviewer 1** Thanks for the suggestions. Our paper covers all well-known utility functions that can be handled within
21 the Eisenberg-Gale framework (i.e. convex, continuous, nonnegative and homogeneous utility functions). See e.g. [9]
22 which lists the class of standard utilities. The only standard ones missing from our paper are CES and Cobb-Douglas,
23 which yield “easy” convex programs (see Appendix A.6). Other utilities do not always yield convex programs.

24 **To Reviewer 2** [It seems straightforward to write down...] In fact, there is an entire literature on convex programming
25 formulations of equilibrium problems. Many papers have been written on the celebrated Eisenberg-Gale convex
26 program, which is a very non-obvious construction. See, e.g., the well-known book “Algorithmic Game Theory.”
27 Furthermore, our contributions are clearly *not* formulation of any convex program or *directly* calling any FOM. [The
28 authors claim that this is one of the contributions...] As stated in the summary of contributions and reiterated above,
29 when solving the convex programs using FOMs, in order to achieve linear convergence, we (need to and did) establish
30 new bounds on various equilibrium quantities (Lemma 1-3). See “To all Reviewers” for other technical contributions
31 along this line. Furthermore, utilizing the structure of problem (4) for QL utilities, Theorem 7 gives a $1/T$ last-iterate
32 convergence rate for Mirror Descent (MD), a result much stronger than the general theory of MD. We also show that it
33 yields highly efficient updates which are also interpretable dynamics (l. 284-287). With these clarifications, we hope
34 that you can reevaluate our contributions.

35 **To Reviewer 3** [“just” applying standard theory] Note that our new bounds on equilibrium quantities (Lemma 1-3)
36 are crucial in establishing linear rates (Thm 5-7). Several other new techniques are needed as explained in “To all
37 Reviewers.” [choice of FOMs arbitrary] We choose them because (i) PG achieves a linear rate due to our theory, (ii)
38 FW solution at iteration t has nt nonzeros (useful when computing a low accuracy solution with very large m), (iii)
39 PR/MD exploits problem structure to get $1/T$ last-iterate convergence, gives desirable dynamics (284-287) and is
40 fast when computing low-accuracy solutions (l. 326). To the best of our knowledge, dual averaging does not have
41 particular theoretical advantages over standard settings. [MD rate on last iterate] Indeed, the convergence result for
42 MD/PR (Thm 7) is nonstandard and specific to the Fisher market setting. It is because the entropy DGF on the prices
43 aligns with the objective to give a form of relative Lipschitz continuity (Lemma 14 in Appendix C.3). [heavy-tailed
44 distribution] Thanks for the suggestion! We ran experiments on Cauchy valuations: PGLS slows down significantly
45 but PR is not affected (across many sizes and accuracy levels). We conjecture that the Hoffman constant, similar to
46 the matrix condition number $\sigma_{\max}/\sigma_{\min}$, degrades significantly upon heavy-tail v (this seems extremely difficult to
47 formalize). In contrast, the bound (Thm 7) for MD/PR is independent of v . Will include in the final version. [In practice
48 ... \tilde{h} ?] Quadratic extrapolations are used in the experiments. We hope that you can reevaluate our contributions given
49 the clarifications.

50 **To Reviewer 4** Many thanks for the detailed, helpful comments. We will take all into account but due to space
51 constraints we respond only to a few here. The reason we do not compare to convex program solvers is that we are
52 motivated by large-scale settings, which conic solvers do not scale to. For smaller problems, open-source solvers such
53 as ECOS are much slower than our methods, while the industry-grade Mosek solver (high-performance C code; our
54 code is in python) outperforms PGLS, but is slower than PR and FW when a low-accuracy solution is needed. We will
55 point this out. In Theorem 1, existence of ME is guaranteed by existence of an EG optimal solution (which is then
56 guaranteed by the Extreme Value Theorem; it is often simply assumed but we will clarify). We do implement Mirror
57 Descent: the PR dynamics (6) are MD applied to the convex program (4) for QL markets.