- 1 We thank all reviewers for their useful feedback and acknowledgement of our contribution. All comments will be
- ² addressed in greater details in the revision. We first answer some common questions brought up by reviewers.
- 3 Numerical illustrations: Thanks for the suggestion! We agree with reviewers that it is useful to provide some numerical
- 4 evidence to illustrate the stability analysis. In Figures (a) and (b), we provide preliminary numerical illustrations of the
- 5 associated ODE models (original affine switching systems, upper and lower comparison systems) of the asynchronous
- 6 Q-learning and the averaging Q-learning on a toy MDP example with with |S| = 2 and |A| = 2. Our simulation
- 7 empirically verifies the theory claimed in the paper. Richer numerical evidence will be included in the revision.



(a) Stability of asynchronous Q-learning

(b) Stability of averaging Q-learning

- 8 Assumptions: Given that this is the first work that bridges switching system theory with RL algorithms, we intentionally
- 9 adopt simplified assumptions (such as i.i.d. assumption, orthogonal feature vectors) to avoid complications. Indeed,
- 10 these assumptions are quite common in many seminal work in RL theory and can be relaxed. We will dedicate a
- 11 discussion section on these assumptions and discuss potential relaxations or limitations.
- 12 Below we address the each reviewer's comments separately.
- **13 Response to Reviewer 1**
- 14 Step-sizes: Using learning rates dependent on state-action observations may be useful in practice for small tabular
- ¹⁵ MDPs; however, for modern developments in RL with function approximation, using learning rates independent of ¹⁶ state-action observations is dominant in the literature.
- ¹⁶ state-action observations is dominant in the interature.
- 17 Finite-sample guarantee: Our current framework only provides asymptotic convergence similar as most work on ODE
- analysis. Recent advances [Srikant & Ying, 2019; Hu & Syed, 2019; Chen et al., 2019; Wang & Giannakis, 2020;
- ¹⁹ Devraj & Meyn , 2020] show promise in the derivation of non-asymptotic convergence rates using more sophisticated
- 20 ODE analysis tools. We leave this extension for future investigation.
- 21 Response to Reviewer 2
- 22 Simulation/Assumptions: See discussions above.
- 23 **Clarity of the paper**: We will improve the presentation of the paper and avoid heavy notations.
- 24 Response to Reviewer 3
- 25 Numerical evidence. See discussions above.
- **Example.** We used Example 1 as a *simple analytical illustration* of the tightness of these sufficient conditions. We agree that it might be too simple to justify the claim. We will consider nontrivial examples with interpretable feature
- ²⁸ matrices and verify these sufficient conditions numerically.
- 29 Response to Reviewer 4
- 30 Global Lipschitz continuity: This is not necessarily an assumption. We show that all the ODE models associated with
- 31 Q-learning and its variants in this paper indeed satisfy the global Lipschitz continuity. Relaxing this condition into
- weaker ones would be promising to accommodate a wider array of RL algorithms, which we will pursue in the future.
- Lemma 3: The spectrum is not well defined for switching systems because the subsystem matrices change. It is known
- that each subsystem matrix having negative spectrum does not guarantee the stability of the overall switching system.
- ³⁵ Lemma 3 is a particular *necessary and sufficient condition* for the stability of the overall switching system.