

We thank the reviewers for the careful feedback and appreciate the time spent reading our paper. Detailed responses are as below:

Literature on online LP (OLP) and the contribution of our work:

(i) From the algorithmic perspective, our algorithm has a strongly polynomial  $O(\text{nnz}(A))$  flop complexity (linear in the number of non-zero entries in  $A$ ), while the previous OLP algorithms all require solving  $O(\log n)$  or  $O(n)$  of LPs (increasing to the full size over time). For example, Agrawal et al. (2014) solved  $O(\log n)$  LPs and Kesselheim et al. (2014) solved  $O(n)$  LPs. As far as we know, the algorithm is the first of its kind and the most efficient OLP algorithm so far.

(ii) As to the analytical framework, we analyze the algorithm under both stochastic input model and random permutation model with minimal technical assumptions. As mentioned by the reviewer, our algorithms share similarity with the network control algorithm in Neely, M.J. (2010), but our analysis extends their analysis (in i.i.d. setting) to the random permutation setting. And the application of Permutational Rademacher Complexity is novel in online learning and regret analysis literature.

Technically speaking, we adopt a different method in analyzing the random permutation model than the works in OLP literature (such as Feldman et al., Agrawal et al., Kesselheim et al. etc.). They all applied a shrinkage technique and its idea is to perform the online learning as if the constraint is shrunk by a factor of  $1-\varepsilon$ , and then the output online solution would be feasible with high probability for the original problem. We provide an alternative treatment in the paper by directly relating the past and future observations (through Permutational Rademacher complexity and other related derivations). Then it enables us to (i) analyze a non-shrinkage version of Agrawal et al. (2014) and (ii) relax the previous non-negativeness assumption.

Breaking constraints: In the updated version of the paper, we will provide an algorithm that is feasible with high probability and the regret of the new algorithm absorbs the current constraint violation into the current regret. The idea is very straightforward: We could randomly select  $O(\sqrt{n})$  of decision variables based on the online solution obtained by the current algorithm and switch them from one to zero. Intuitively, the switching operation will reduce the total constraint consumption so that the new solution is provably feasible based on a concentration argument. In this sense, the bi-objective performance measure in our paper is more for a better exposition but not entailed by our analytical framework.

Besides, the notion of “Artificially created randomness” refers to the application of our algorithm in solving integer LPs. The original inputs of an integer LP might not satisfy the i.i.d. assumption. When this is the case, we can random permute the inputs of the integer LP to make it satisfy the random permutation assumption, and then our algorithm and analysis can be applied. In this way, the randomness can be artificially created when it is not inherent. We also appreciate the typos pointed out by the reviewers. We will correct these typos and include more numerical experiments in the updated paper. We will highlight (i) the effectiveness in large-scale problems and (ii) our fast solution as a warm start for other exact LP solvers.