

1 We thank the reviewers for their careful reading and their constructive comments. We clarify the main points raised in  
 2 the reviews:

3 **Rev#1 1.1 More experiments:** With suggestion from Rev.#2, we propose to replace the randomly generated  $x$  in  
 4 Figure.6(c/d) by images from the digits dataset. The curves do not change significantly but this use-case is close to a  
 5 real application. Also, such method could be used to accelerate the resolution of inverse problems (e.g. see Adler et al  
 6 2017). We will emphasize such application in our introduction.

7 **Rev#2 2.1 Comparison with backtracking (BT):** BT typically considers (e.g. Nesterov 2013) candidate steps of the  
 8 form  $\alpha_0 \beta^k, k \geq 0$ , where  $\alpha_0$  is an initial guess and  $\beta < 1$  is a shrinking constant (Armijo line-search). It is indeed  
 9 close to our work with two main differences: i) At test time, BT should be performed at each step of ISTA, making the  
 10 cost of one iteration larger than ISTA/SLISTA. Also note that the hyperparameters  $\alpha_0, \beta$  are critical and hard to tune  
 11 properly. ii) BT finds greedily a suitable step-size at a given iteration for a given sample. The goal with SLISTA is  
 12 to learn a sequence of step-sizes for all iterations jointly for the input distribution. We will clarify this relationship  
 13 and core differences in the text and replace Fig.2 with Fig A.2, where we show the results of a perfect line-search  
 14 algorithm (taking at each iteration a step-size exactly minimizing the loss function), and two backtracking with different  
 15 parameters  $(\alpha_0, \beta)$ . One could also use BT to learn steps for the whole distribution, by picking step sizes in the set  
 16  $(\alpha_0 \beta^k)_{k \geq 0}$ . This corresponds to a discretized version of the SLISTA problem, which we feel is out of the scope of the  
 17 current article.

18 **2.2 Results relative to [2]:** We stress that Theorem 4.4 states that LISTA only learns step-sizes *asymptotically*. Thus,  
 19 LISTA can still leverage the dictionary structure in the first layers and improve compared to SLISTA, as seen in  
 20 Fig.6(a/c). Our results are consistent with those of [2], as their section 2.3.1 shows a phase transition where the structure  
 21 of  $D$  leveraged by FacNet only improves the convergence in the first layers.

22 **2.3 Clarify proposition 4.1:** The proof will be rewritten as the part about uniform convergence of ISTA is in-  
 23 deed confusing. ISTA's rate of convergence writes:  $F_x(z_T(x)) - F_x^* \leq \frac{L}{T} \|z^*(x)\|_2^2$ . Further,  $\|z^*\|_2 \leq \|z^*\|_1 \leq$   
 24  $\frac{1}{\lambda} F_x^* \leq \frac{1}{\lambda} F_x(0) = \frac{1}{\lambda} \|x\|_2^2$ . Since  $\mathcal{B}_\infty$  is bounded,  $\|z^*(x)\|_2$  is uniformly bounded. It yields an inequality of the  
 25 form  $F_x^* \leq F_x(z_T(x)) \leq F_x^* + K/T$  where  $K$  is independent of  $x$ . Taking expectations shows as advertised:  
 26  $\mathbb{E}_{x \sim p}[F_x(\Phi_{\Theta_{\text{ISTA}}^{(T)}}(x))] \xrightarrow{T \rightarrow \infty} \mathbb{E}_{x \sim p}[F_x^*]$ . This proposition shows the global (theoretical) minimizer of the unsupervised  
 27 problem solves the LASSO. In practice, as any neural network, the computed solution faces optimization (non-convexity)  
 28 and generalization (empirical risk minimization) errors.

29 **2.4 Fig.4:** We modify Fig.4 by adding the  $2/L_S$  line (see Fig A.1). Learned steps are mainly included in  $]0, 2/L_S[$   
 30 which guarantees the cost function decrease, if the support inclusion condition is verified. However, steps above  $2/L_S$   
 31 may lead to greater decrease of the loss function as seen in Fig A.2.

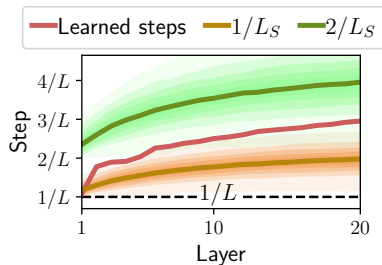
32 **2.5 Semi-real experiment:** see 1.1.

33 **Rev#3 3.1 Effect of different step-sizes:** Fig.2 shows that larger step-sizes can lead to faster convergence. See 2.1.

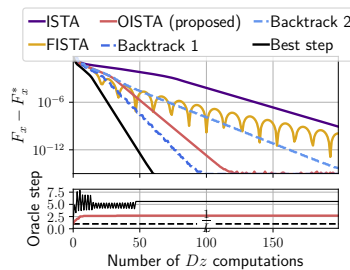
34 **3.2 Is SLISTA better? ALISTA does not work** In the experiments, we see that SLISTA is better when the iterates are  
 35 very sparse (high  $\lambda$ ), leveraging the same properties as OISTA. It is outperformed by LISTA for small  $\lambda$ . As stated in  
 36 the text (1.229/252), following thm.4.4., ALISTA cannot converge in this unsupervised setting.

37 **3.3 Unsupervised/Supervised:** ISTA converges to a solution of the Lasso. In the supervised setting, a unique solution  
 38 exists independently from the Lasso solution. In most practical cases, the Lasso solution and the supervised solution are  
 39 different. Even if they match, it is for a specific  $\lambda$ , unknown *a priori*. Hence ISTA does not converge for the supervised  
 40 problem. This is typically highlighted in Figure.1(a) from the ALISTA article (Liu et al 2019) where the MSE of ISTA  
 41 plateaus. We will clarify this in the text.

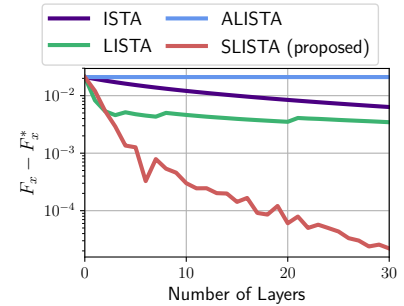
42 **3.4 Extension to convolutional cases/robustness:** This is surely an interesting extension, which cannot unfortunately  
 43 be inserted in the paper for lack of space. Also, the theoretical results from ALISTA paper were previously published in  
 44 NeurIPS without the convolution/robustness part (Chen et al 2018).



(A.1) Step-sizes learned with SLISTA



(A.2) Performance of F/ISTA, OISTA and ISTA with Backtracking / oracle line search.



(A.3) Performances of SLISTA on digits with  $\lambda = 0.8$ .