

| Dataset                            |       |        | OSSN             | RKO              | SVCM             | BCOP                    |
|------------------------------------|-------|--------|------------------|------------------|------------------|-------------------------|
| CIFAR10<br>( $\epsilon = 36/255$ ) | Small | Clean  | 61.83 $\pm$ 0.86 | 61.75 $\pm$ 0.57 | 61.88 $\pm$ 0.52 | <b>64.25</b> $\pm$ 0.39 |
|                                    |       | Robust | 47.77 $\pm$ 0.74 | 47.39 $\pm$ 0.44 | 47.17 $\pm$ 0.41 | <b>49.95</b> $\pm$ 0.17 |
|                                    | Large | Clean  | 68.28 $\pm$ 0.51 | 69.47 $\pm$ 0.24 | 69.44 $\pm$ 0.26 | <b>72.01</b> $\pm$ 0.24 |
|                                    |       | Robust | 54.26 $\pm$ 0.40 | 55.41 $\pm$ 0.21 | 53.57 $\pm$ 0.18 | <b>58.26</b> $\pm$ 0.17 |

1 We thank all the reviewers for their thorough feedback. We have updated all tables to report results from 5 repeated  
2 trials. Our reported improvements are consistent throughout. We would like to re-emphasize our main contributions  
3 here: **(1)** To the best of our knowledge, we are the first to reveal the disconnectedness of the space of orthogonal  
4 convolutions. We believe our analysis demonstrates this space is unexpectedly complicated and inherently difficult to  
5 optimize over. **(2)** We analyze and identify the shortcomings of existing methods of enforcing Lipschitz-constrained  
6 convolutions. In particular, we find gradient attenuation to be a common problem among many of these methods and  
7 propose using orthogonal convolutions to circumvent this. **(3)** We adapt Xiao et al. [40]\*’s orthogonal convolution  
8 initialization procedure to be used for optimizing over the orthogonal convolution space. Our parameterization alleviates  
9 the issues of the disconnected orthogonal convolution space that arose in our analysis. We verified its effectiveness on  
10 adversarial robustness and Wasserstein distance estimation tasks over the pre-existing methods.

11 **Reviewer 1:** All empirical results now include error bars (see example table, top). We observed statistical significance  
12 throughout. We discuss other points below.

13 *Quality - (1)* Our BCOP parameterization lies in the space of orthogonal convolutions, which is 1-Lipschitz only under  
14 the  $L_2$  metric. We will make clear that we focus on Lipschitz convolutional networks with the  $L_2$  metric only.

15 *Quality - (2a)* To clarify, the statement was trying to demonstrate a relationship between the gradient norm before and  
16 after back-propagating through a 1-Lipschitz function. To be precise, let  $\mathbf{y} = f(\mathbf{x})$  for some 1-Lipschitz  $f$ , and  $\mathcal{L}(\mathbf{y})$   
17 be a loss function. We have  $\left\| \frac{\partial \mathcal{L}}{\partial \mathbf{x}} \right\|_2 = \left\| \frac{\partial \mathcal{L}}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \right\|_2 \leq \left\| \frac{\partial \mathcal{L}}{\partial \mathbf{y}} \right\|_2 \left\| \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \right\|_2 \leq \text{Lip}(f) \left\| \frac{\partial \mathcal{L}}{\partial \mathbf{y}} \right\|_2 = \left\| \frac{\partial \mathcal{L}}{\partial \mathbf{y}} \right\|_2$ , where  $\left\| \frac{\partial \mathcal{L}}{\partial \mathbf{x}} \right\|_2$  and  
18  $\left\| \frac{\partial \mathcal{L}}{\partial \mathbf{y}} \right\|_2$  are the input and output gradient norm correspondingly, and  $\text{Lip}(f)$  is the Lipschitz constant of the function  $f$ .  
19 We will adjust the phrasing of the statement in the paper to include this detailed explanation.

20 *Quality - (2b, 2c, 2d)* Gouk et al. [15]\*<sup>1</sup> write that OSSN will “project it back to the closest matrix in the feasible  
21 set measured by the matrix distance metric induced by taking the operator norm”. One can prove that OSSN is a  
22 valid projection under 2-norm but not Frobenius norm (while SV clipping is a valid projection for both norms as R1  
23 suggested). Because OSSN uses a different norm for the steepest descent direction and the projection step, it’s not  
24 guaranteed to converge; we give a counterexample in Section A of the supplemental material.

25 *Quality - (2e)* By “reshaping a kernel into a matrix”, we were referring to **flattening** a 2-D convolution kernel tensor of  
26 shape  $(c_o, c_i, k, k)$  into a  $c_o \times c_i k^2$  matrix, where  $c_i, c_o, k$  are input channel size, output channel size and kernel size,  
27 respectively; whereas, the **matrix form** of the convolution operator is a  $hwc_o \times hwc_i$  matrix ( $h, w$  are the input/output  
28 spatial dimensions). Tsuzuku et al. [36]\* has shown a constant factor of the spectral norm of the “**reshaped/flattened**  
29 **kernel**” bounds the Lipschitz constant of the convolution operator, arising from the repetitions of each convolution  
30 kernel tensor element in the **matrix form** due to overlapping convolution windows.

31 *Quality - (2f)* The optimal dual function must have gradient norm 1 almost everywhere on the support (see Corollary 1 in  
32 Gemicic et al. [41]), which can be achieved by gradient norm preservation throughout the network. However, we did not  
33 mean to imply that limiting the function space we are optimizing over to be gradient norm preserving is theoretically the  
34 best way to estimate Wasserstein distance. We will adjust the writing and supply the additional references accordingly.

35 *Clarity* As pointed out by the reviewer, “Lipschitz network” indeed refers to a network with a specified Lipschitz  
36 constant that is enforced tightly. This will be clarified. We will re-organize method and experiment sections to clarify  
37 notations and key experimental details.

38 **Reviewer 2** expressed concerns over novelty of this work. As discussed above, we do not simply combine methods  
39 from Xiao et al. [40]\* and Anil et al. [1]\*. Xiao et al. [40]\*’s algorithm is used for initializing orthogonal convolutions  
40 while we need to parameterize the orthogonal convolution space to be optimized over. Moreover, our theoretical analysis  
41 enables our BCOP parameterization to be configured to maximize the expressiveness of the orthogonal convolution.

42 **Reviewer 3** inquired about run-time comparison of our Lipschitz convolutional network against standard non-Lipschitz  
43 convolutional network. During the training of the “large” architecture described in the paper,<sup>2</sup> the BCOP-parameterized  
44 network takes 0.138 seconds per training iteration while a standard non-Lipschitz network with the same architecture  
45 only takes 0.041 seconds per training iteration. As for the other Lipschitz methods, RKO takes 0.120 seconds and  
46 OSSN (with one power iteration) takes 0.113 seconds. We will report these values in the paper.

47 **Additional Reference:** [41] Mevlana Gemicic, Zeynep Akata, and Max Welling. Primal-dual Wasserstein GAN. 2018.

<sup>1</sup>The starred references are from the original paper. Any additional references are provided below.

<sup>2</sup>Training speed benchmark setup: CIFAR10, NVIDIA P100, batch size of 128.