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# The Infinite Mixture of Infinite Gaussian Mixtures

## *Supplementary Document*

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## 1 Data Set Characteristics

Table 1: Characteristics of the five data sets compared in terms of the number of clusters ( $K$ ), the number of instances ( $n$ ), the data dimensionality ( $d$ ), the ratio of smallest to largest cluster size ( $r$ ), skewedness, and multi-modality.

Data set	K	n	d	r	skewed	multi-modal
Flower	4	17,000	2	0.22	no	yes
Lymphoma	2-4	1,856-24,564	5	0.002-0.270	yes	no
Rare classes	3	279,546	6	0.0003	yes	yes
Hyperspectral	8	21,518	30	0.04	no	yes
Letter recognition	26	20,000	16	0.96	unknown	unknown

## 2 Number of Clusters Predicted

Table 2: Number of clusters generated by  $I^2GMM$ ,  $I^2GMMp$ , VB, KD-VB, and ColGibbs on the five data sets. Predicted numbers shown are modes of ten repetitions. Clusters with less than ten instances are ignored. For the lymphoma data set the minimum and maximum of the predicted number of clusters across thirty sub-data sets are reported.

Data set	$I^2GMM$ $I^2GMMp$ VB KD-VB ColGibbs					
	True	Predicted				
Flower	4	4	4	10	8	12
Lymphoma	2-4	2-9	2-9	17-53	2-8	5-22
Rare Classes	3	29	30	60	11	65
Hyperspectral	8	42	41	9	4	91
Letter Recognition	26	46	47	93	18	153

### 3 Results for the Thirty Sub-data Sets in the Lymphoma Data Set

Table 3: Micro  $F_1$  scores produced by the four techniques on each of the thirty sub-data of the lymphoma data set. Numbers in parenthesis indicate standard deviations across ten repetitions.

Sub-data set	I <sup>2</sup> GMM	I <sup>2</sup> GMMp	VB	KD-VB	Gibbs
1	0.853 (0.005)	0.853 (0.004)	0.502 (0.043)	0.690	0.705 (0.046)
2	0.994 (0.001)	0.994 (0.001)	0.341 (0.048)	0.980	0.551 (0.062)
3	0.971 (0.037)	0.969 (0.029)	0.501 (0.087)	0.926	0.823 (0.016)
4	0.834 (0.024)	0.834 (0.025)	0.490 (0.047)	0.802	0.588 (0.046)
5	0.956 (0.001)	0.955 (0.002)	0.547 (0.049)	0.947	0.827 (0.008)
6	0.984 (0.002)	0.984 (0.002)	0.402 (0.044)	0.985	0.626 (0.089)
7	0.949 (0.008)	0.951 (0.008)	0.553 (0.062)	0.722	0.627 (0.013)
8	0.909 (0.009)	0.914 (0.007)	0.436 (0.042)	0.670	0.611 (0.046)
9	0.946 (0.003)	0.947 (0.002)	0.622 (0.097)	0.934	0.847 (0.025)
10	0.968 (0.002)	0.967 (0.003)	0.540 (0.058)	0.941	0.662 (0.028)
11	0.792 (0.102)	0.856 (0.070)	0.597 (0.056)	0.864	0.874 (0.015)
12	0.868 (0.022)	0.867 (0.026)	0.542 (0.059)	0.803	0.709 (0.042)
13	0.859 (0.002)	0.859 (0.001)	0.441 (0.045)	0.752	0.578 (0.039)
14	0.855 (0.023)	0.854 (0.021)	0.360 (0.051)	0.770	0.573 (0.059)
15	0.888 (0.015)	0.890 (0.011)	0.277 (0.031)	0.690	0.435 (0.017)
16	0.953 (0.013)	0.955 (0.017)	0.421 (0.076)	0.920	0.740 (0.050)
17	0.876 (0.012)	0.862 (0.009)	0.424 (0.053)	0.743	0.607 (0.032)
18	0.922 (0.013)	0.916 (0.019)	0.391 (0.056)	0.747	0.517 (0.024)
19	0.990 (0.001)	0.990 (0.001)	0.491 (0.091)	0.975	0.682 (0.058)
20	0.967 (0.014)	0.964 (0.015)	0.443 (0.051)	0.785	0.592 (0.047)
21	0.945 (0.010)	0.945 (0.008)	0.429 (0.046)	0.806	0.627 (0.019)
22	0.956 (0.009)	0.958 (0.008)	0.423 (0.093)	0.713	0.616 (0.039)
23	0.962 (0.002)	0.962 (0.002)	0.456 (0.043)	0.744	0.632 (0.019)
24	0.948 (0.007)	0.951 (0.010)	0.551 (0.055)	0.892	0.666 (0.017)
25	0.916 (0.025)	0.907 (0.030)	0.421 (0.048)	0.730	0.559 (0.010)
26	0.936 (0.010)	0.932 (0.018)	0.494 (0.054)	0.806	0.641 (0.044)
27	0.871 (0.049)	0.866 (0.064)	0.365 (0.024)	0.828	0.480 (0.024)
28	0.977 (0.009)	0.971 (0.015)	0.295 (0.059)	0.907	0.440 (0.040)
29	0.944 (0.004)	0.946 (0.004)	0.465 (0.061)	0.723	0.632 (0.030)
30	0.814 (0.034)	0.838 (0.033)	0.392 (0.039)	0.782	0.551 (0.026)
Avg.	0.920 (0.016)	0.922 (0.020)	0.454 (0.056)	0.819	0.634 (0.034)

Table 4: Macro  $F_1$  scores produced by the four techniques on each of the thirty sub-data of the lymphoma data set. Numbers in parenthesis indicate standard deviations across ten repetitions.

Sub-data set	I <sup>2</sup> GMM	I <sup>2</sup> GMMp	VB	KD-VB	ColGibbs
1	0.840 (0.005)	0.842 (0.004)	0.504 (0.041)	0.707	0.712 (0.039)
2	0.967 (0.005)	0.969 (0.004)	0.552 (0.032)	0.856	0.697 (0.040)
3	0.963 (0.066)	0.977 (0.016)	0.635 (0.085)	0.932	0.856 (0.016)
4	0.670 (0.024)	0.671 (0.024)	0.461 (0.034)	0.640	0.551 (0.030)
5	0.702 (0.003)	0.705 (0.013)	0.559 (0.042)	0.792	0.765 (0.037)
6	0.971 (0.002)	0.972 (0.002)	0.465 (0.030)	0.957	0.716 (0.055)
7	0.855 (0.024)	0.856 (0.020)	0.563 (0.059)	0.602	0.592 (0.012)
8	0.897 (0.007)	0.903 (0.009)	0.444 (0.043)	0.687	0.628 (0.036)
9	0.938 (0.004)	0.939 (0.002)	0.607 (0.092)	0.926	0.848 (0.034)
10	0.978 (0.003)	0.976 (0.005)	0.616 (0.047)	0.911	0.719 (0.017)
11	0.618 (0.080)	0.671 (0.055)	0.529 (0.048)	0.677	0.756 (0.021)
12	0.797 (0.035)	0.795 (0.042)	0.505 (0.039)	0.747	0.684 (0.029)
13	0.677 (0.011)	0.678 (0.008)	0.561 (0.046)	0.621	0.610 (0.066)
14	0.831 (0.021)	0.834 (0.022)	0.532 (0.032)	0.716	0.649 (0.030)
15	0.784 (0.021)	0.777 (0.025)	0.345 (0.035)	0.655	0.489 (0.010)
16	0.898 (0.033)	0.905 (0.049)	0.581 (0.033)	0.809	0.759 (0.020)
17	0.787 (0.019)	0.768 (0.015)	0.384 (0.036)	0.635	0.532 (0.026)
18	0.744 (0.042)	0.738 (0.013)	0.482 (0.035)	0.775	0.643 (0.014)
19	0.892 (0.007)	0.893 (0.009)	0.536 (0.043)	0.784	0.707 (0.029)
20	0.933 (0.015)	0.933 (0.014)	0.431 (0.037)	0.730	0.553 (0.042)
21	0.896 (0.023)	0.895 (0.018)	0.479 (0.044)	0.815	0.596 (0.032)
22	0.887 (0.031)	0.914 (0.026)	0.518 (0.049)	0.814	0.704 (0.029)
23	0.916 (0.002)	0.916 (0.003)	0.515 (0.043)	0.704	0.658 (0.023)
24	0.921 (0.010)	0.927 (0.015)	0.508 (0.043)	0.811	0.634 (0.022)
25	0.862 (0.037)	0.848 (0.039)	0.417 (0.044)	0.751	0.531 (0.012)
26	0.909 (0.015)	0.904 (0.026)	0.524 (0.064)	0.798	0.665 (0.048)
27	0.850 (0.025)	0.850 (0.037)	0.400 (0.035)	0.771	0.507 (0.020)
28	0.881 (0.004)	0.834 (0.092)	0.572 (0.039)	0.953	0.656 (0.019)
29	0.794 (0.011)	0.801 (0.009)	0.513 (0.054)	0.595	0.608 (0.021)
30	0.740 (0.056)	0.728 (0.043)	0.523 (0.027)	0.700	0.643 (0.044)
Avg.	0.847 (0.021)	0.847 (0.022)	0.509 (0.044)	0.762	0.656 (0.029)

Table 5: Number of clusters generated by the four techniques on each of the thirty sub-data of the lymphoma data set. Predicted numbers shown are modes of ten repetitions.

Sub-data set		I <sup>2</sup> GMM	I <sup>2</sup> GMMp	VB	KD-VB	ColGibbs
	True	Predicted				
1	2	3	3	22	4	6
2	2	3	3	29	3	5
3	2	2	3	20	3	7
4	3	2	2	21	3	8
5	3	2	2	24	4	6
6	2	3	3	28	3	7
7	3	3	3	17	5	7
8	2	3	3	27	5	8
9	2	3	3	22	3	5
10	2	3	3	21	2	8
11	3	2	3	23	3	5
12	4	5	4	35	7	13
13	4	4	4	35	6	13
14	3	6	7	53	7	22
15	3	6	7	46	5	15
16	3	5	4	31	6	10
17	4	5	5	44	5	19
18	3	4	4	40	6	10
19	3	3	3	28	3	9
20	3	4	5	37	7	14
21	3	4	4	40	6	12
22	3	3	4	38	5	10
23	4	5	5	36	8	13
24	4	8	7	48	5	21
25	3	5	6	48	8	19
26	3	5	5	42	8	15
27	3	9	8	44	6	19
28	2	3	2	41	5	9
29	3	5	5	25	7	12
30	4	6	4	33	6	13