

Appendix for paper: Structured Learning via Logistic Regression

Theorem 5. *The difference of l and l_1 is bounded by*

$$l_1(x, y, F) \leq l(x, y, F) \leq l_1(x, y, F) + \epsilon H_{\max}, \quad H_{\max} = \sum_{\alpha} \log |y_{\alpha}|.$$

Proof. Defining $\mu^* = \arg \max_{\mu \in \mathcal{M}} \theta \cdot \mu$ and $\mu' = \arg \max_{\mu \in \mathcal{M}} \theta \cdot \mu + \epsilon \sum_{\alpha} H(\mu_{\alpha})$, one can write

$$\begin{aligned} l(x, y; F) - l_1(x, y; F) &= -F(x, y) + \max_{\mu \in \mathcal{M}} \left(\theta \cdot \mu + \sum_{\alpha} \epsilon H(\mu_{\alpha}) \right) + F(x, y) - \max_{\mu \in \mathcal{M}} \theta \cdot \mu \\ &= \max_{\mu \in \mathcal{M}} \left(\theta \cdot \mu + \sum_{\alpha} \epsilon H(\mu_{\alpha}) \right) - \max_{\mu \in \mathcal{M}} \theta \cdot \mu \\ &= \theta \cdot \mu' - \theta \cdot \mu^* + \sum_{\alpha} \epsilon H(\mu'_{\alpha}) \\ &\leq \epsilon \sum_{\alpha} \log |y_{\alpha}|. \end{aligned}$$

The last line follows from the fact that $\theta \cdot \mu^* \geq \theta \cdot \mu'$, and that $H(\mu'_{\alpha}) \leq \log |y_{\alpha}|$. \square

| Denoising | | | | | | Horses | | | | | |
|--|------|--------|--------|--------|------|--|------|--------|--------|--------|------|
| $\mathcal{F}_i \setminus \mathcal{F}_{ij}$ | Zero | Const. | Linear | Boost. | MLP | $\mathcal{F}_i \setminus \mathcal{F}_{ij}$ | Zero | Const. | Linear | Boost. | MLP |
| Zero | .490 | .490 | .490 | .441 | .490 | Zero | .211 | .211 | .212 | .209 | .210 |
| Const. | .490 | .490 | .490 | .440 | .490 | Const. | .211 | .211 | .212 | .209 | .210 |
| Linear | .443 | .077 | .059 | .048 | .033 | Linear | .141 | .139 | .126 | .105 | .113 |
| Boost. | .429 | .032 | .014 | .008 | .008 | Boost. | .074 | .068 | .063 | .057 | .060 |
| MLP | .435 | .031 | .014 | .008 | .008 | MLP | .054 | .051 | .046 | .039 | .041 |

Table 2: Univariate Training Error Rates

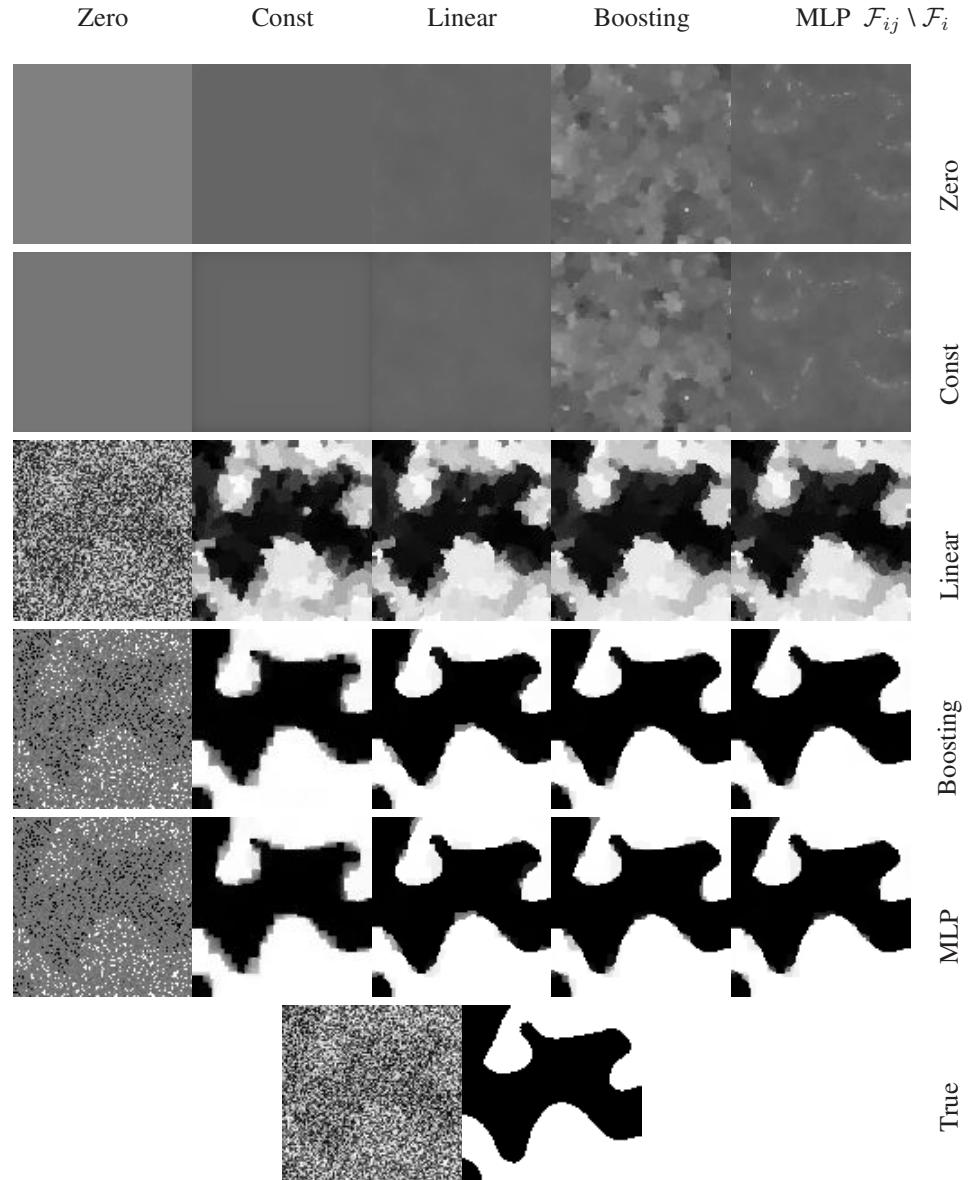


Figure 4: Example Predictions on the Denoising Dataset

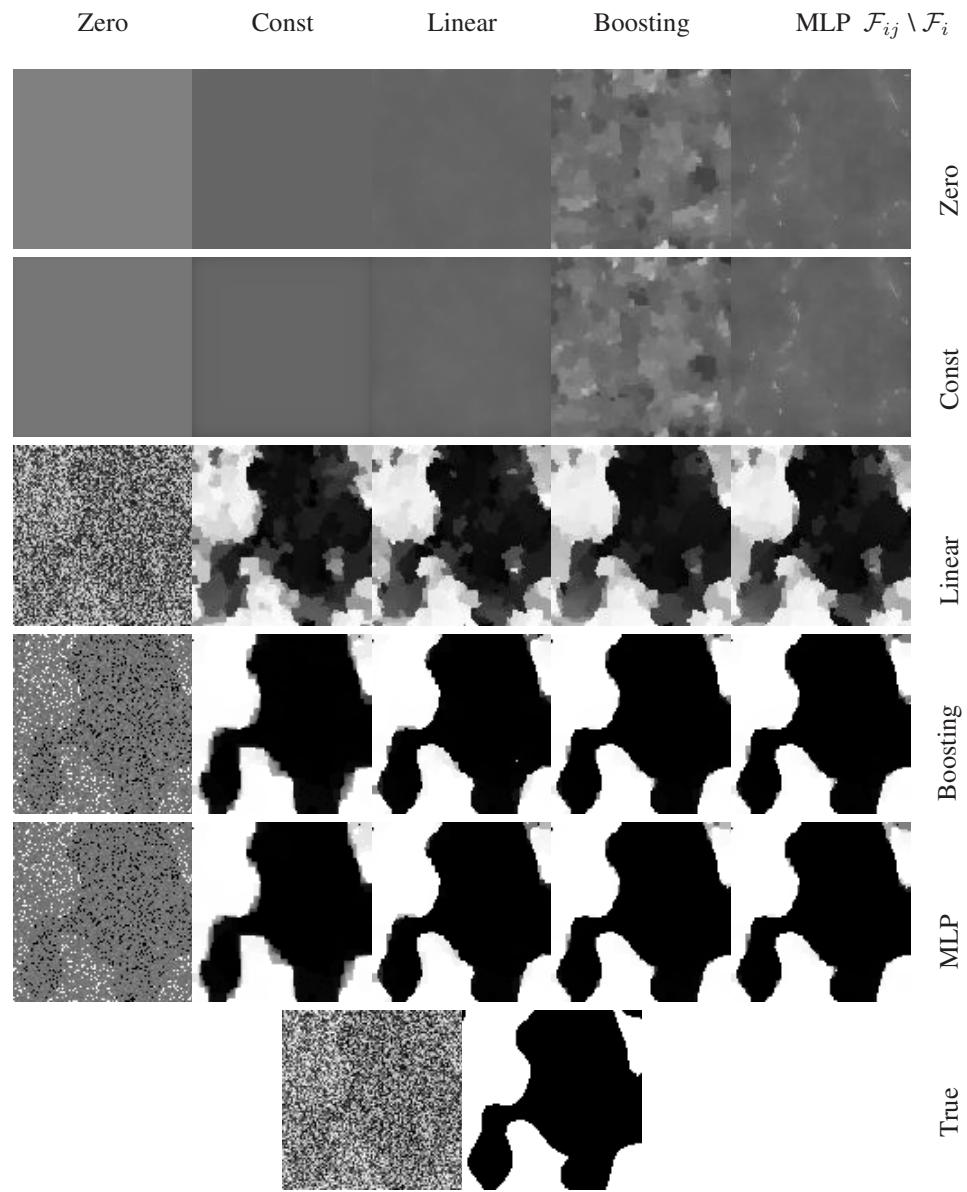


Figure 5: Example Predictions on the Denoising Dataset

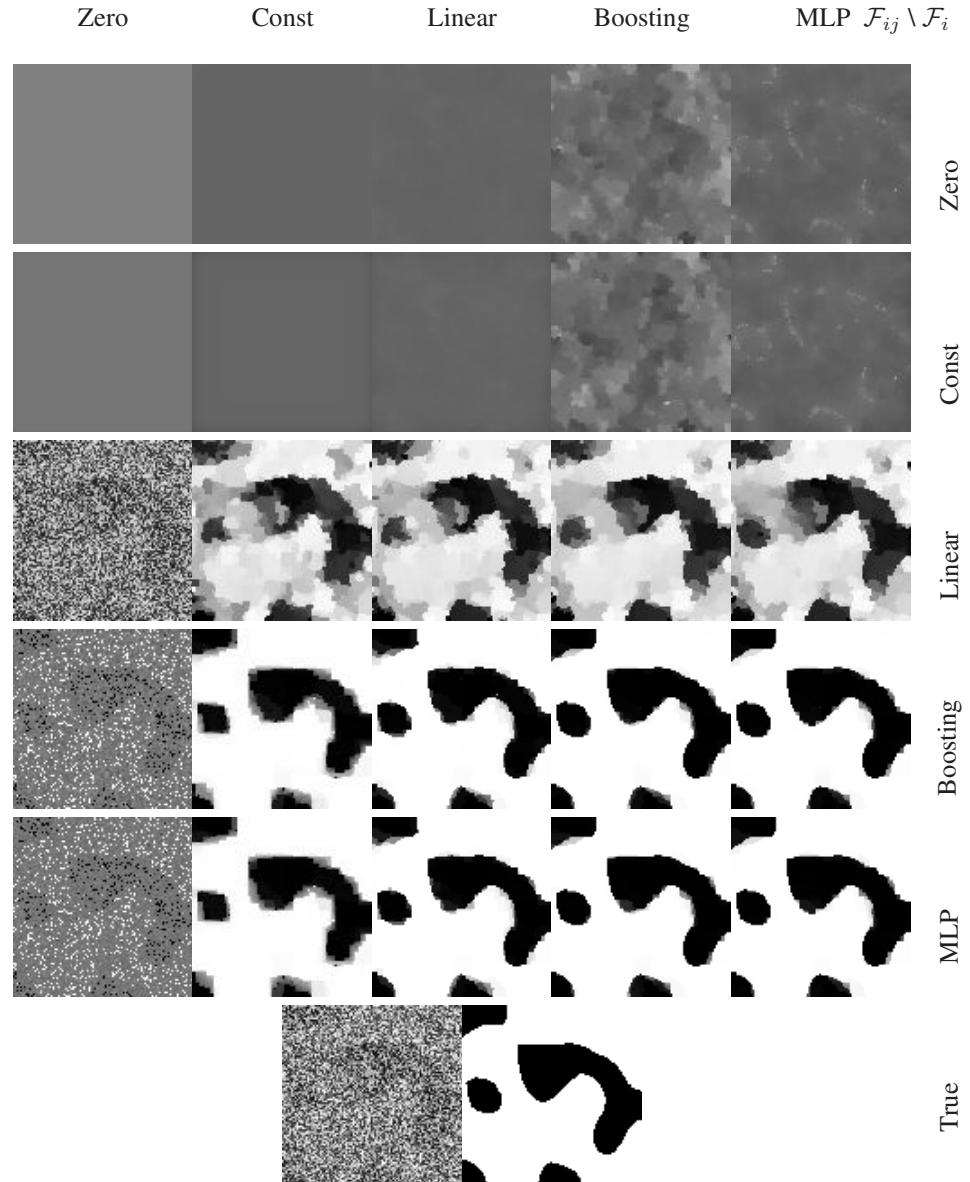


Figure 6: Example Predictions on the Denoising Dataset

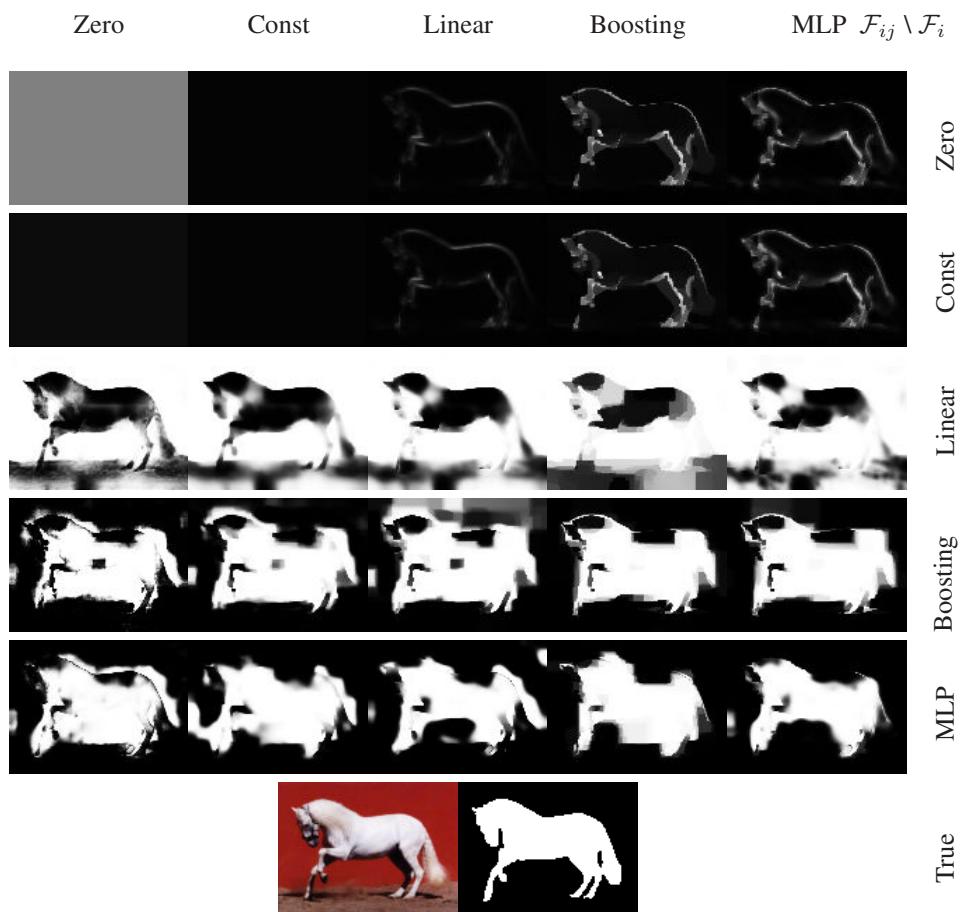


Figure 7: Example Predictions on the Horses Dataset

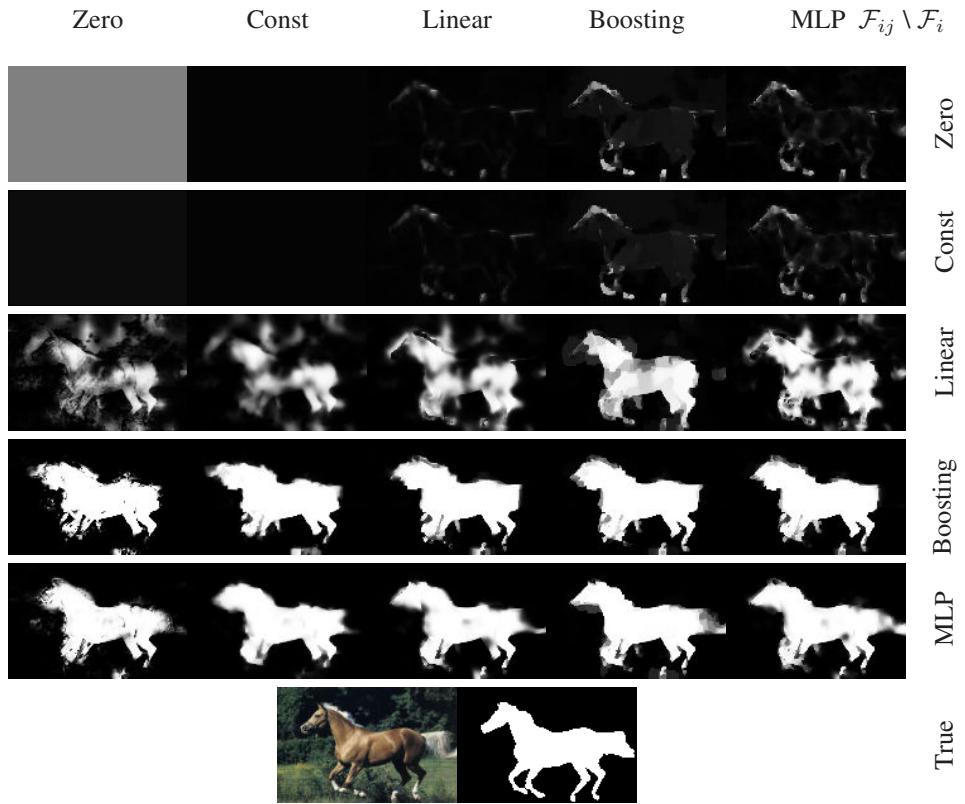


Figure 8: Example Predictions on the Horses Dataset

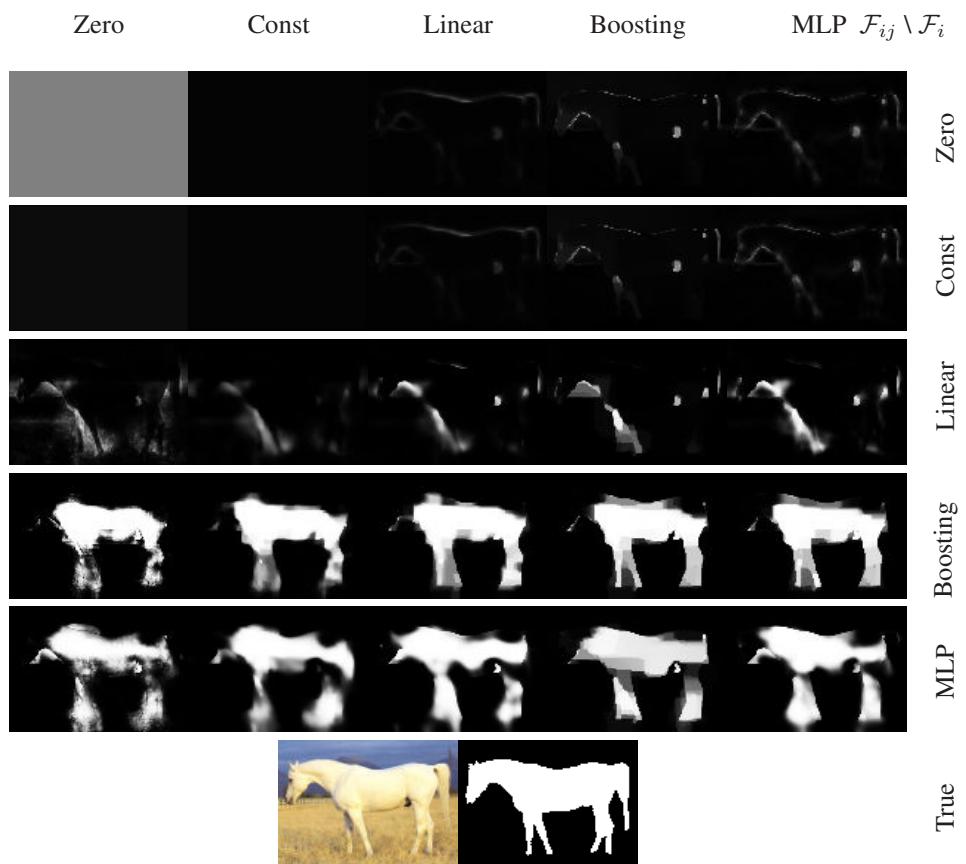


Figure 9: Example Predictions on the Horses Dataset

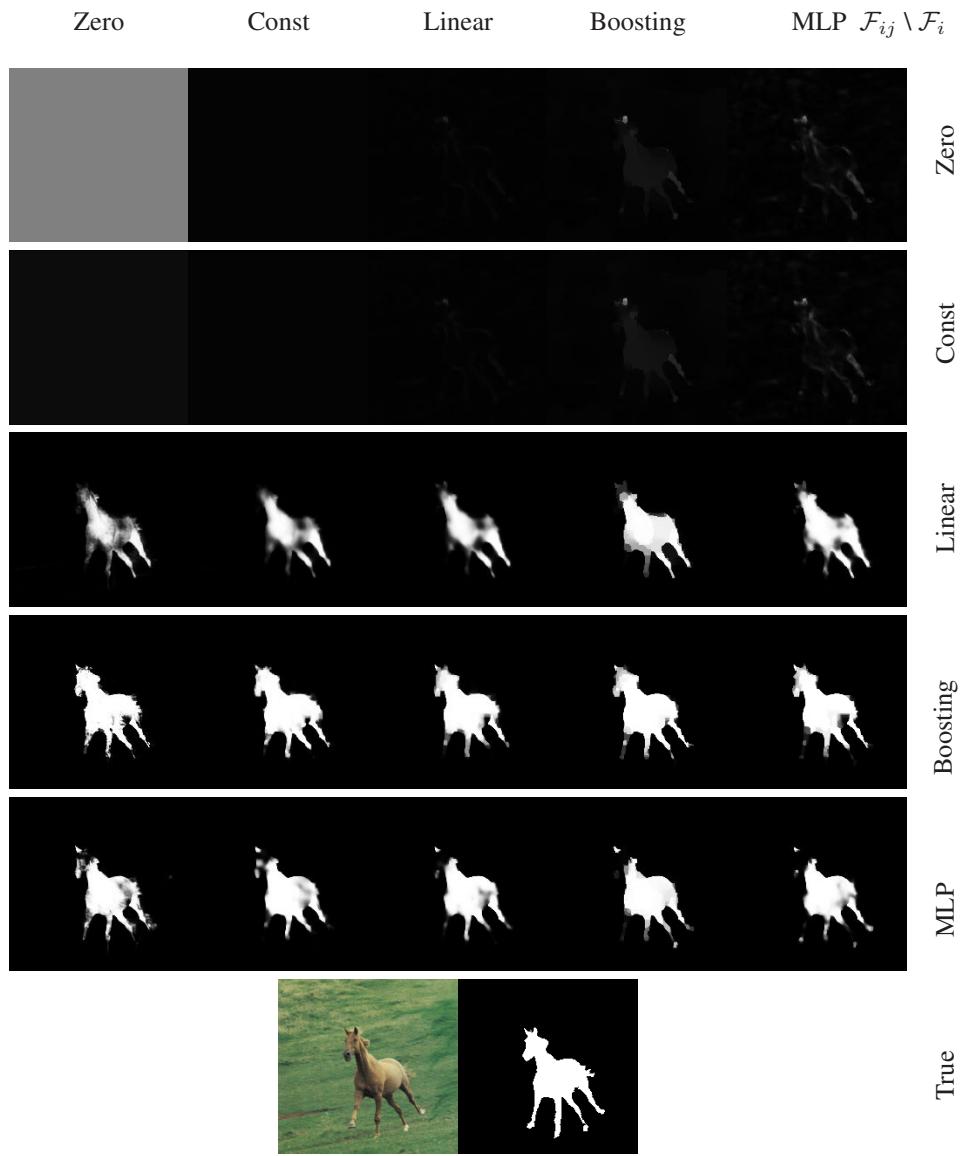


Figure 10: Example Predictions on the Horses Dataset

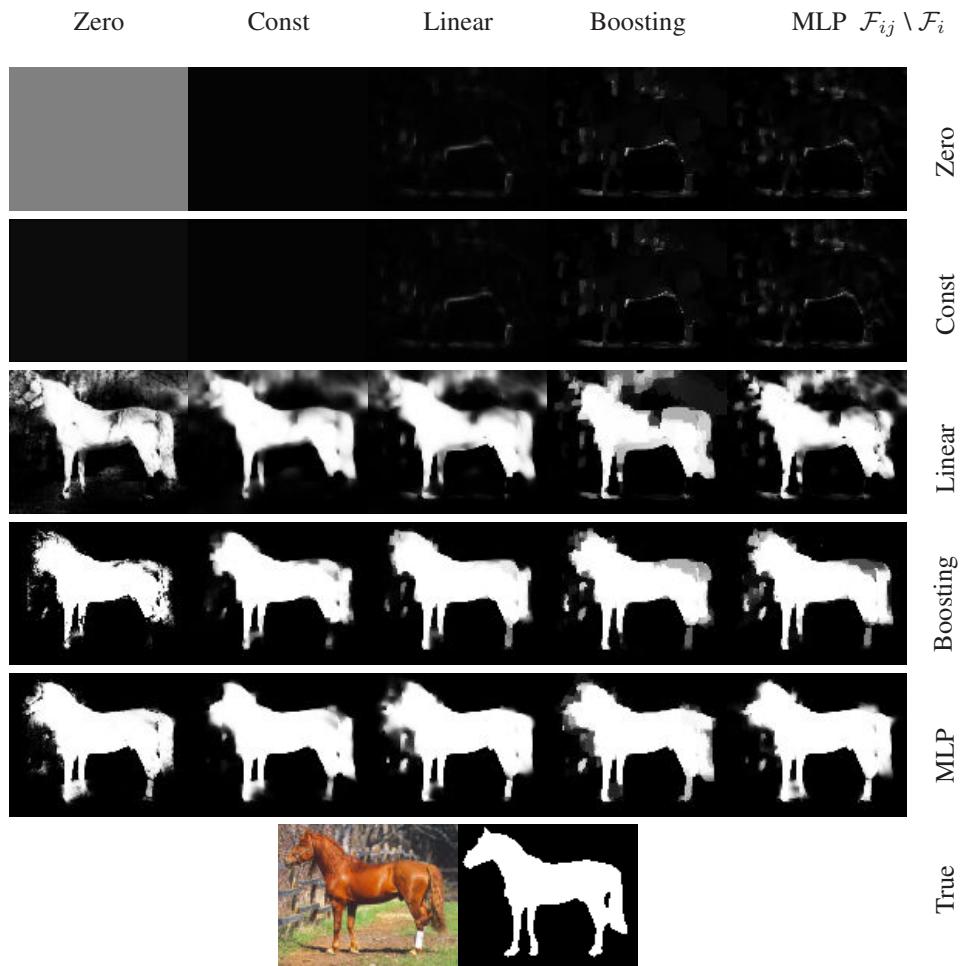


Figure 11: Example Predictions on the Horses Dataset

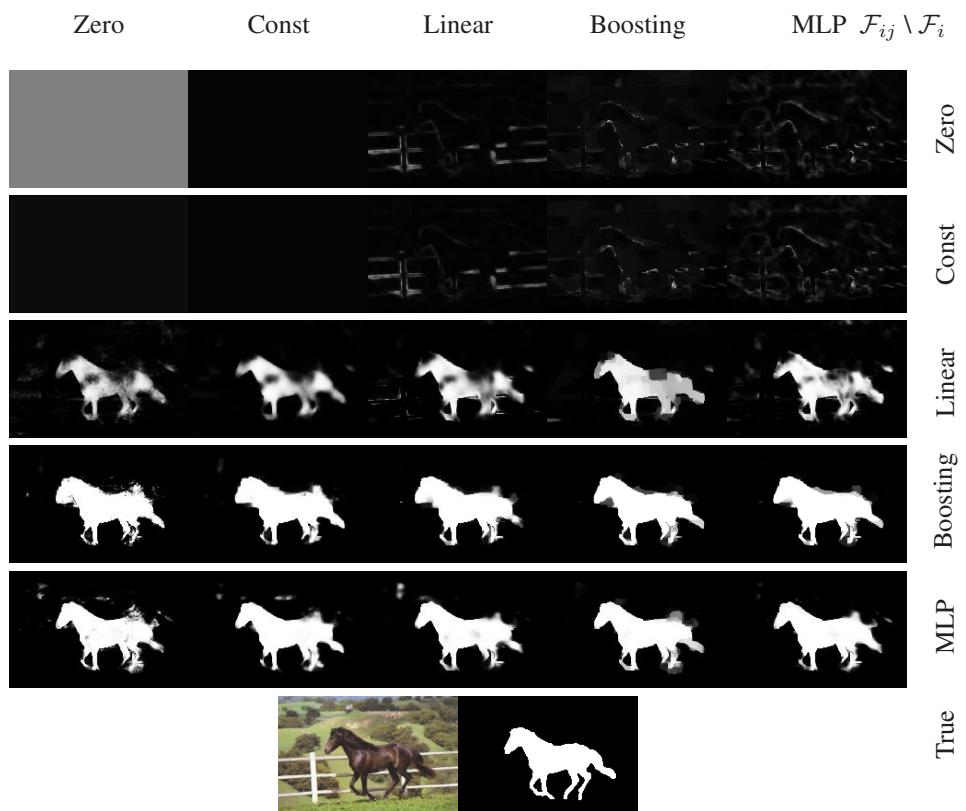


Figure 12: Example Predictions on the Horses Dataset