A Proof of Lemma 3

Proof. From the strict convexity of $J(\eta)$ it follows that $J'(\eta)$ has positive derivative for all η . Hence, $J'(\eta)$ is invertible. From the symmetry of $J(\eta)$,

$$J'(\eta) = -J'(1-\eta)$$

and, for any v such that $\eta = [J']^{-1}(v)$,

$$\begin{aligned} v &= -J'(1-[J']^{-1}(v)) \\ [J']^{-1}(-v) &= 1-[J']^{-1}(v). \end{aligned}$$

□

B Proof of Theorem 4

Proof. Given that $C_\phi(\eta, f)$ is a canonical risk and (16), the loss function of (14) can be simplified to

$$\begin{aligned} \phi(v) &= -J[f^{-1}(v)] - (1-f^{-1}(v))J'[f^{-1}(v)] \\ &= -J\{[J']^{-1}(v)\} - (1-[J']^{-1}(v))J'\{[J']^{-1}(v)\} \\ &= -J\{[J']^{-1}(v)\} - (1-[J']^{-1}(v))v. \end{aligned}$$

The proof follows from taking derivatives on both sides,

$$\begin{aligned} \phi'(v) &= -J'\{[J']^{-1}(v)\}\{[J']^{-1}\}'(v) - (1-[J']^{-1}(v)) + \{[J']^{-1}\}'(v)v \\ &= -v\{[J']^{-1}\}'(v) - (1-[J']^{-1}(v)) + \{[J']^{-1}\}'(v)v \\ &= -(1-[J']^{-1}(v)) \\ &= -[J']^{-1}(-v), \end{aligned}$$

where we have also used (15). Furthermore, using (16),

$$\begin{aligned} \phi'(v) &= -(1-[J']^{-1}(v)) && (32) \\ &= -(1-[f^*]^{-1}(v)) && (33) \\ &= [f^*]^{-1}(v) - 1. && (34) \end{aligned}$$

□