On-line Policy Improvement using Monte-Carlo Search

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Abstract

We present a Monte-Carlo simulation algorithm for real-time policy improvement of an adaptive controller. In the Monte-Carlo simulation, the long-term expected reward of each possible action is statistically measured, using the initial policy to make decisions in each step of the simulation. The action maximizing the measured expected reward is then taken, resulting in an improved policy. Our algorithm is easily parallelizable and has been implemented on the IBM SP1 and SP2 parallel-RISC supercomputers.

We have obtained promising initial results in applying this algorithm to the domain of backgammon. Results are reported for a wide variety of initial policies, ranging from a random policy to TD-Gammon, an extremely strong multi-layer neural network. In each case, the Monte-Carlo algorithm gives a substantial reduction, by as much as a factor of 5 or more, in the error rate of the base players. The algorithm is also potentially useful in many other adaptive control applications in which it is possible to simulate the environment.

1 INTRODUCTION

Policy iteration, a widely used algorithm for solving problems in adaptive control, consists of repeatedly iterating the following policy improvement computation (Bertsekas, 1995): (1) First, a value function is computed that represents the long-term expected reward that would be obtained by following an initial policy. (This may be done in several ways, such as with the standard dynamic programming algorithm.) (2) An improved policy is then defined which is greedy with respect to that value function. Policy iteration is known to have rapid and robust convergence properties, and for Markov tasks with lookup-table state-space representations, it is guaranteed to convergence to the optimal policy.

In typical uses of policy iteration, the policy improvement step is an extensive off-line procedure. For example, in dynamic programming, one performs a sweep through all states in the state space. Reinforcement learning provides another approach to policy improvement; recently, several authors have investigated using RL in conjunction with nonlinear function approximators to represent the value functions and/or policies (Tesauro, 1992; Crites and Barto, 1996; Zhang and Dietterich, 1996). These studies are based on following actual state-space trajectories rather than sweeps through the full state space, but are still too slow to compute improved policies in real time. Such function approximators typically need extensive off-line training on many trajectories before they achieve acceptable performance levels.

In contrast, we propose an on-line algorithm for computing an improved policy in real time. We use Monte-Carlo search to estimate $V_P(x, a)$, the expected value of performing action a in state x and subsequently executing policy P in all successor states. Here, P is some given arbitrary policy, as defined by a "base controller" (we do not care how P is defined or was derived; we only need access to its policy decisions). In the Monte-Carlo search, many simulated trajectories starting from (x, a) are generated following P, and the expected long-term reward is estimated by averaging the results from each of the trajectories. (Note that Monte-Carlo sampling is needed only for non-deterministic tasks, because in a deterministic task, only one trajectory starting from (x, a) would need to be examined.) Having estimated $V_P(x, a)$, the improved policy P' at state x is defined to be the action which produced the best estimated value in the Monte-Carlo simulation, i.e., $P'(x) = \arg\max_a V_P(x, a)$.

1.1 EFFICIENT IMPLEMENTATION

The proposed Monte-Carlo algorithm could be very CPU-intensive, depending on the number of initial actions that need to be simulated, the number of time steps per trial needed to obtain a meaningful long-term reward, the amount of CPU per time step needed to make a decision with the base controller, and the total number of trials needed to make a Monte-Carlo decision. The last factor depends on both the variance in expected reward per trial, and on how close the values of competing candidate actions are.

We propose two methods to address the potentially large CPU requirements of this approach. First, the power of parallelism can be exploited very effectively. The algorithm is easily parallelized with high efficiency: the individual Monte-Carlo trials can be performed independently, and the combining of results from different trials is a simple averaging operation. Hence there is relatively little communication between processors required in a parallel implementation.

The second technique is to continually monitor the accumulated Monte-Carlo statistics during the simulation, and to prune away both candidate actions that are sufficiently unlikely (outside some user-specified confidence bound) to be selected as the best action, as well as candidates whose values are sufficiently close to the value of the current best estimate that they are considered equivalent (i.e., choosing either would not make a significant difference). This technique requires more communication in a parallel implementation, but offers potentially large savings in the number of trials needed to make a decision.

2 APPLICATION TO BACKGAMMON

We have initially applied the Monte-Carlo algorithm to making move decisions in the game of backgammon. This is an absorbing Markov process with perfect state-space information, and one has a perfect model of the nondeterminism in the system, as well as the mapping from actions to resulting states.

In backgammon parlance, the expected value of a position is known as the "equity" of the position, and estimating the equity by Monte-Carlo sampling is known as performing a "rollout." This involves playing the position out to completion many times with different random dice sequences, using a fixed policy P to make move decisions for both sides. The sequences are terminated at the end of the game (when one side has borne off all 15 checkers), and at that time a signed outcome value (called "points") is recorded. The outcome value is positive if one side wins and negative if the other side wins, and the magnitude of the value can be either 1, 2, or 3, depending on whether the win was normal, a gammon, or a backgammon. With normal human play, games typically last on the order of 50-60 time steps. Hence if one is using the Monte-Carlo player to play out actual games, the Monte-Carlo trials will on average start out somewhere in the middle of a game, and take about 25-30 time steps to reach completion.

In backgammon there are on average about 20 legal moves to consider in a typical decision. The candidate plays frequently differ in expected value by on the order of .01. Thus in order to resolve the best play by Monte-Carlo sampling, one would need on the order of 10K or more trials per candidate, or a total of hundreds of thousands of Monte-Carlo trials to make one move decision. With extensive statistical pruning as discussed previously, this can be reduced to several tens of thousands of trials. Multiplying this by 25-30 decisions per trial with the base player, we find that about a million base-player decisions have to be made in order to make one Monte-Carlo decision. With typical human tournament players taking about 10 seconds per move, we need to parallelize to the point that we can achieve at least 100K base-player decisions per second.

Our Monte-Carlo simulations were performed on the IBM SP1 and SP2 parallel-RISC supercomputers at IBM Watson and at Argonne National Laboratories. Each SP node is equivalent to a fast RS/6000, with floating-point capability on the order of 100 Mflops. Typical runs were on configurations of 16-32 SP nodes, with parallel speedup efficiencies on the order of 90%.

We have used a variety of base players in our Monte-Carlo simulations, with widely varying playing abilities and CPU requirements. The weakest (and fastest) of these is a purely random player. We have also used a few single-layer networks (i.e., no hidden units) with simple encodings of the board state, that were trained by back-propagation on an expert data set (Tesauro, 1989). These simple networks also make fast move decisions, and are much stronger than a random player, but in human terms are only at a beginner-to-intermediate level. Finally, we used some multi-layer nets with a rich input representation, encoding both the raw board state and many hand-crafted features, trained on self-play using the TD(λ) algorithm (Sutton, 1988; Tesauro, 1992). Such networks play at an advanced level, but are too slow to make Monte-Carlo decisions in real time based on full rollouts to completion. Results for all these players are presented in the following two sections.

2.1 RESULTS FOR SINGLE-LAYER NETWORKS

We measured the game-playing strength of three single-layer base players, and of the corresponding Monte-Carlo players, by playing several thousand games against a common benchmark opponent. The benchmark opponent was TD-Gammon 2.1 (Tesauro, 1995), playing on its most basic playing level (1-ply search, i.e., no lookahead). Table 1 shows the results. Lin-1 is a single-layer neural net with only the raw board description (number of White and Black checkers at each location) as

Network	Base player	Monte-Carlo player	Monte-Carlo CPU
Lin-1	-0.52 ppg	-0.01 ppg	5 sec/move
Lin-2	-0.65 ppg	-0.02 ppg	5 sec/move
Lin-3	-0.32 ppg	+0.04 ppg	10 sec/move

Table 1: Performance of three simple linear evaluators, for both initial base players and corresponding Monte-Carlo players. Performance is measured in terms of expected points per game (ppg) vs. TD-Gammon 2.1 1-ply. Positive numbers indicate that the player here is better than TD-Gammon. Base player stats are the results of 30K trials (std. dev. about .005), and Monte-Carlo stats are the results of 5K trials (std. dev. about .02). CPU times are for the Monte-Carlo player running on 32 SP1 nodes.

input. Lin-2 uses the same network structure and weights as Lin-1, plus a significant amount of random noise was added to the evaluation function, in order to deliberately weaken its playing ability. These networks were highly optimized for speed, and are capable of making a move decision in about 0.2 msec on a single SP1 node. Lin-3 uses the same raw board input as the other two players, plus it has a few additional hand-crafted features related to the probability of a checker being hit; there is no noise added. This network is a significantly stronger player, but is about twice as slow in making move decisions.

We can see in Table 1 that the Monte-Carlo technique produces dramatic improvement in playing ability for these weak initial players. As base players, Lin-1 should be regarded as a bad intermediate player, while Lin-2 is substantially worse and is probably about equal to a human beginner. Both of these networks get trounced by TD-Gammon, which on its 1-ply level plays at strong advanced level. Yet the resulting Monte-Carlo players from these networks appear to play about equal to TD-Gammon 1-ply. Lin-3 is a significantly stronger player, and the resulting Monte-Carlo player appears to be clearly better than TD-Gammon 1-ply. It is estimated to be about equivalent to TD-Gammon on its 2-ply level, which plays at a strong expert level.

The Monte-Carlo benchmarks reported in Table 1 involved substantial amounts of CPU time. At 10 seconds per move decision, and 25 move decisions per game, playing 5000 games against TD-Gammon required about 350 hours of 32-node SP machine time. We have also developed an alternative testing procedure, which is much less expensive in CPU time, but still seems to give a reasonably accurate measure of performance strength. We measure the average equity loss of the Monte-Carlo player on a suite of test positions. We have a collection of about 800 test positions, in which every legal play has been extensively rolled out by TD-Gammon 2.1 1-ply. We then use the TD-Gammon rollout data to grade the quality of a given player's move decisions.

Test set results for the three linear evaluators, and for a random evaluator, are displayed in Table 2. It is interesting to note for comparison that the TD-Gammon 1-ply base player scores 0.0120 on this test set measure, comparable to the Lin-1 Monte-Carlo player, while TD-Gammon 2-ply base player scores 0.00843, comparable to the Lin-3 Monte-Carlo player. These results are exactly in line with what we measured in Table 1 using full-game benchmarking, and thus indicate that the test-set methodology is in fact reasonably accurate. We also note that in each case, there is a huge error reduction of potentially a factor of 4 or more in using the Monte-Carlo technique. In fact, the rollouts summarized in Table 2 were done using fairly aggressive statistical pruning; we expect that rolling out decisions more

Evaluator	Base loss	Monte-Carlo loss	Ratio
Random	0.330	0.131	2.5
Lin-1	0.040	0.0124	3.2
Lin-2	0.0665	0.0175	3.8
Lin-3	0.0291	0.00749	3.9

Table 2: Average equity loss per move decision on an 800-position test set, for both initial base players and corresponding Monte-Carlo players. Units are ppg; smaller loss values are better. Also computed is ratio of base player loss to Monte-Carlo loss.

extensively would give error reduction ratios closer to factor of 5, albeit at a cost of increased CPU time.

2.2 RESULTS FOR MULTI-LAYER NETWORKS

Using large multi-layer networks to do full rollouts is not feasible for real-time move decisions, since the large networks are at least a factor of 100 slower than the linear evaluators described previously. We have therefore investigated an alternative Monte-Carlo algorithm, using so-called "truncated rollouts." In this technique trials are not played out to completion, but instead only a few steps in the simulation are taken, and the neural net's equity estimate of the final position reached is used instead of the actual outcome. The truncated rollout algorithm requires much less CPU time, due to two factors: First, there are potentially many fewer steps per trial. Second, there is much less variance per trial, since only a few random steps are taken and a real-valued estimate is recorded, rather than many random steps and an integer final outcome. These two factors combine to give at least an order of magnitude speed-up compared to full rollouts, while still giving a large error reduction relative to the base player.

Table 3 shows truncated rollout results for two multi-layer networks: TD-Gammon 2.1 1-ply, which has 80 hidden units, and a substantially smaller network with the same input features but only 10 hidden units. The first line of data for each network reflects very extensive rollouts and shows quite large error reduction ratios, although the CPU times are somewhat slower than acceptable for real-time play. (Also we should be somewhat suspicious of the 80 hidden unit result, since this was the same network that generated the data being used to grade the Monte-Carlo players.) The second line of data shows what happens when the rollout trials are cut off more aggressively. This yields significantly faster run-times, at the price of only slightly worse move decisions.

The quality of play of the truncated rollout players shown in Table 3 is substantially better than TD-Gammon 1-ply or 2-ply, and it is also substantially better than the full-rollout Monte-Carlo players described in the previous section. In fact, we estimate that the world's best human players would score in the range of 0.005 to 0.006 on this test set, so the truncated rollout players may actually be exhibiting superhuman playing ability, in reasonable amounts of SP machine time.

3 DISCUSSION

On-line search may provide a useful methodology for overcoming some of the limitations of training nonlinear function approximators on difficult control tasks. The idea of using search to improve in real time the performance of a heuristic controller

Hidden Units	Base loss	Truncated Monte-Carlo loss	Ratio	M-C CPU
10	0.0152	0.00318 (11-step, thorough)	4.8	25 sec/move
		0.00433 (11-step, optimistic)	3.5	9 sec/move
80	0.0120	0.00181 (7-step, thorough)	6.6	65 sec/move
		0.00269 (7-step, optimistic)	4.5	18 sec/move

Table 3: Truncated rollout results for two multi-layer networks, with number of hidden units and rollout steps as indicated. Average equity loss per move decision on an 800-position test set, for both initial base players and corresponding Monte-Carlo players. Again, units are ppg, and smaller loss values are better. Also computed is ratio of base player loss to Monte-Carlo loss. CPU times are for the Monte-Carlo player running on 32 SP1 nodes.

is an old one, going back at least to (Shannon, 1950). Full-width search algorithms have been extensively studied since the time of Shannon, and have produced tremendous success in computer games such as chess, checkers and Othello. Their main drawback is that the required CPU time increases exponentially with the depth of the search, i.e., $T \sim B^D$, where B is the effective branching factor and D is the search depth. In contrast, Monte-Carlo search provides a tractable alternative for doing very deep searches, since the CPU time for a full Monte-Carlo decision only scales as $T \sim N \cdot B \cdot D$, where N is the number of trials in the simulation.

In the backgammon application, for a wide range of initial policies, our on-line Monte-Carlo algorithm, which basically implements a single step of policy iteration, was found to give very substantial error reductions. Potentially 80% or more of the base player's equity loss can be eliminated, depending on how extensive the Monte-Carlo trials are. The magnitude of the observed improvement is surprising to us: while it is known theoretically that each step of policy iteration produces a strict improvement, there are no guarantees on how much improvement one can expect. We have also noted a rough trend in the data: as one increases the strength of the base player, the ratio of error reduction due to the Monte-Carlo technique appears to increase. This could reflect superlinear convergence properties of policy iteration.

In cases where the base player employs an evaluator that is able to estimate expected outcome, the truncated rollout algorithm appears to offer favorable tradeoffs relative to doing full rollouts to completion. While the quality of Monte-Carlo decisions is not as good using truncated rollouts (presumably because the neural net's estimates are biased), the degradation in quality is fairly small in at least some cases, and is compensated by a great reduction in CPU time. This allows more sophisticated (and thus slower) base players to be used, resulting in decisions which appear to be both better and faster.

The Monte-Carlo backgammon program as implemented on the SP offers the potential to achieve real-time move decision performance that exceeds human capabilities. In future work, we plan to augment the program with a similar Monte-Carlo algorithm for making doubling decisions. It is quite possible that such a program would be by far the world's best backgammon player.

Beyond the backgammon application, we conjecture that on-line Monte-Carlo search may prove to be useful in many other applications of reinforcement learning and adaptive control. The main requirement is that it should be possible to simulate the environment in which the controller operates. Since basically all of the recent successful applications of reinforcement learning have been based on training in simulators, this doesn't seem to be an undue burden. Thus, for example, Monte-

Carlo search may well improve decision-making in the domains of elevator dispatch (Crites and Barto, 1996) and job-shop scheduling (Zhang and Dietterich, 1996).

We are additionally investigating two techniques for training a controller based on the Monte-Carlo estimates. First, one could train each candidate position on its computed rollout equity, yielding a procedure similar in spirit to TD(1). We expect this to converge to the same policy as other TD(λ) approaches, perhaps more efficiently due to the decreased variance in the target values as well as the easily parallelizable nature of the algorithm. Alternately, the base position – the initial position from which the candidate moves are being made – could be trained with the best equity value from among all the candidates (corresponding to the move chosen by the rollout player). In contrast, TD(λ) effectively trains the base position with the equity of the move chosen by the base controller. Because the improved choice of move achieved by the rollout player yields an expectation closer to the true (optimal) value, we expect the learned policy to differ from, and possibly be closer to optimal than, the original policy.

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