Simple Spin Models for the Development of Ocular Dominance Columns and Iso-Orientation Patches

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Abstract

Simple classical spin models well-known to physicists as the ANNNI and Heisenberg XY Models, in which long-range interactions occur in a pattern given by the Mexican Hat operator, can generate many of the structural properties characteristic of the ocular dominance columns and iso-orientation patches seen in cat and primate visual cortex.

1 INTRODUCTION

In recent years numerous models for the formation of ocular dominance columns (Malsburg, 1979 ; Swindale, 1980; Miller, Keller, & Stryker, 1989) and of iso-orientation patches (Malsburg 1973; Swindale 1982 & Linsker 1986)have been published. Here we show that simple spin models can reproduce many of the observed features. Our work is similar to, but independent of a recent study employing spin models (Tanaka, 1990).

1.1 OCULAR DOMINANCE COLUMNS

We use a one-dimensional classical spin Hamiltonian on a two-dimensional lattice with long-range interactions. Let σ_i be a spin vector restricted to the orientations $\uparrow$ and $\downarrow$ in the lattice space, and let the spin Hamiltonian be:

$$H_{OD} = -\sum_i \sum_{j \neq i} w_{ij}\, \sigma_i \bullet \sigma_j \,, \tag{1}$$

where w_{ij} is the well-known "Mexican Hat" distribution of weights:

$$w_{ij} = a_+ \exp(-|i-j|^2/\sigma_+^2) - a_- \exp(-|i-j|^2/\sigma_-^2) \tag{2}$$

with $\sigma_+ < \sigma_-$ and $a_+ / a_- = \sigma_-^2/\sigma_+^2$. Evidently $\sigma_i \bullet \sigma_j = \pm|\sigma_i||\sigma_j| = \pm 1$, so that

$$H_{OD} = -\sum_i \sum_{j \neq i} w_{ij}^{s} - \sum_i \sum_{j \neq i} w_{ij}^{o} \tag{3}$$

where $w_{ij}^{s} = w_{ij}$ if $\sigma_i = \sigma_j$, and $w_{ij}^{o} = -w_{ij}$ if $\sigma_i \neq \sigma_j$.

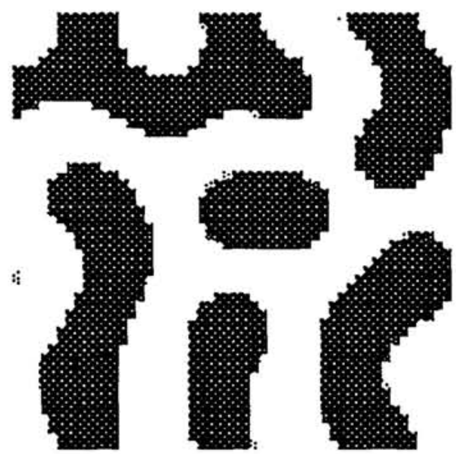

Figure 1. Pattern of Ocular Dominance which results from simulated annealing of the energy function H_{OD}. Light and dark shadings correspond respectively to the two eyes.

Let s denote retinal fibers from the same eye and o fibers from the opposite eye. Then H_{OD} represents the "energy" of interactions between fibers from the two eyes. It is relatively easy to find a configuration of spins which minimizes H_{OD} by simulated annealing (Kirkpatrick, Gelatt & Vecchi 1983). The result is shown in figure 1. It will be seen that the resulting pattern of right and left eye spins σ^R and σ^L is disordered, but at a constant wavelength determined in large part by the space constants σ_+ and σ_-.

Breaking the symmetry of the initial conditions (or letting the lattivce grow systematically) results in ordered patterns.

If H_{OD} is considered to be the energy function of a network of spins exhibiting gradient dynamics (Hirsch & Smale, 1974), then one can write equations for the evolution of spin patterns in the form:

$$\frac{d}{dt}\sigma_i^{\alpha} = -\frac{\partial}{\partial\sigma_i^{\alpha}} H_{OD} = \sum_{j\neq i} w_{ij}^{\alpha\beta}\sigma_j^{\beta}$$

$$= \sum_{j\neq i} w_{ij}^{s}\sigma_i^{\alpha} + \sum_{j\neq i} w_{ij}^{o}\sigma_i^{\beta} = \sum_{j\neq i} w_{ij}\sigma_i^{\alpha} - \sum_{j\neq i} w_{ij}\sigma_i^{\beta}, \qquad (4)$$

where α = R or L, β = L or R respectively. Equation (4) will be recognized as that proposed by Swindale in 1979.

1. 2 ISO-ORIENTATION PATCHES

Now let σ_i represent avector in the plane of the lattice which runs continuously from $\uparrow$ to $\downarrow$ without reference to eye class. It follows that

$$\sigma_i \cdot \sigma_j = |\sigma_i|\,|\sigma_i| \cos(\theta_i - \theta_j) \qquad (5)$$

where θ_i is the orientation of the ith spin vector. The appropriate classical spin Hamiltonian is:

$$H_{IO} = -\sum_i \sum_{j\neq i} w_{ij}\,\sigma_i \cdot \sigma_j = -\sum_i \sum_{j\neq i} w_{ij}\,|\sigma_i|\,|\sigma_i| \cos(\theta_i - \theta_j). \qquad (6)$$

Physicists will recognize H_{OD} as a form of the Ising Lattice Hamiltonian with long-range alternating next nearest neighbor interactions, a type of ANNNI model (Binder, 1986) and H_{IO} as a similar form of the Heisenberg XY Model for antiferromagnetic materials (Binder 1986).

Again one can find a spin configuration that minimizes H_{IO} by simulated annealing. The result is shown in figure 2 in which six differing orientations are depicted, corresponding to 30^o increments (note that $\theta + \pi$ is equivalent to θ). It will be seen that there are long stretches of continuously changing spin vector orientations, with intercalated discontinuities and both clockwise and counter-clockwise singular regions around which the orientations rotate. A one-dimensional slice shows some of these features, and is shown in figure 3.

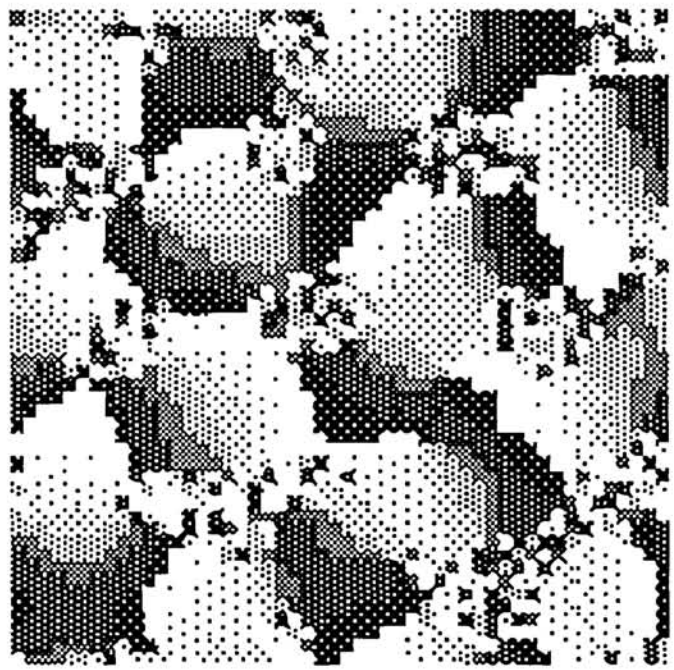

Figure 2. Pattern of orientation patches obtained by simulated annealing of the energy function H_{IO}. Six differing orientations varying from 0^o to 180^o are represented by the different shadings.

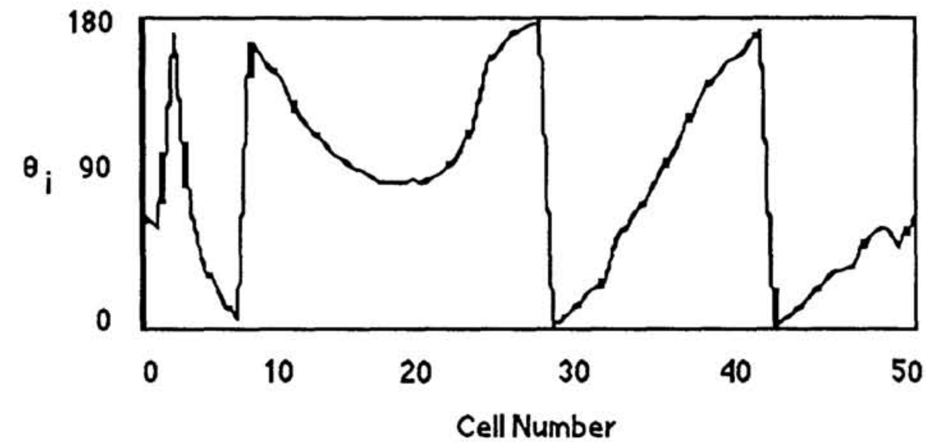

Figure 3. Details of a one-dimensional slice through the orientation map. Long stretches of smoothly changing orientations are evident.

The <u>length</u> of σ_i is also correlated with these details. Figure 4 shows that $|\sigma_i|$ is large in smoothly changing regions and smallest in the neighborhood of a singularity. In fact this model reproduces most of the details of iso-orientation patches found by Blasdel and Salama (1986).

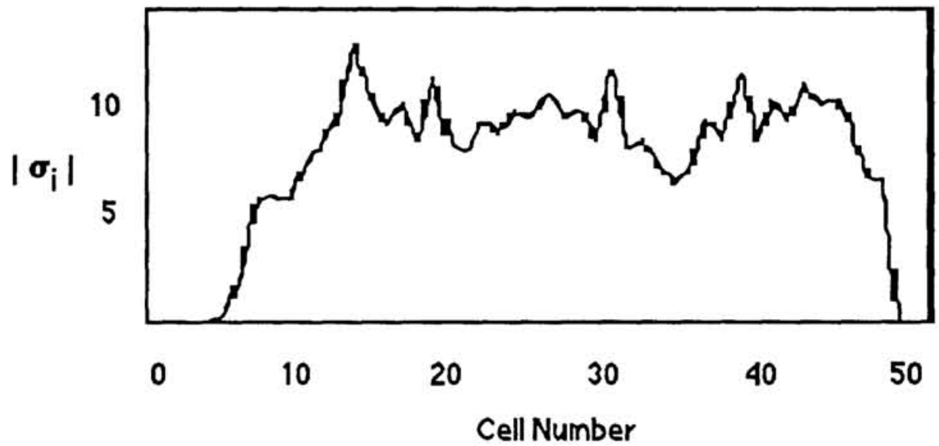

Figure 4. Variation of $|\sigma_i|$ along the same one-dim. slice through the orientation map shown in figure 3. The amplitude drops only near singular regions.

For example, the change in orientation per unit length, $|\mathrm{grad}\theta_i|$ is shown in figure 5. It will be seen that the lattice is "tiled", just as in the data from visual cortex, with max $|\mathrm{grad}\theta_i|$ located at singularities.

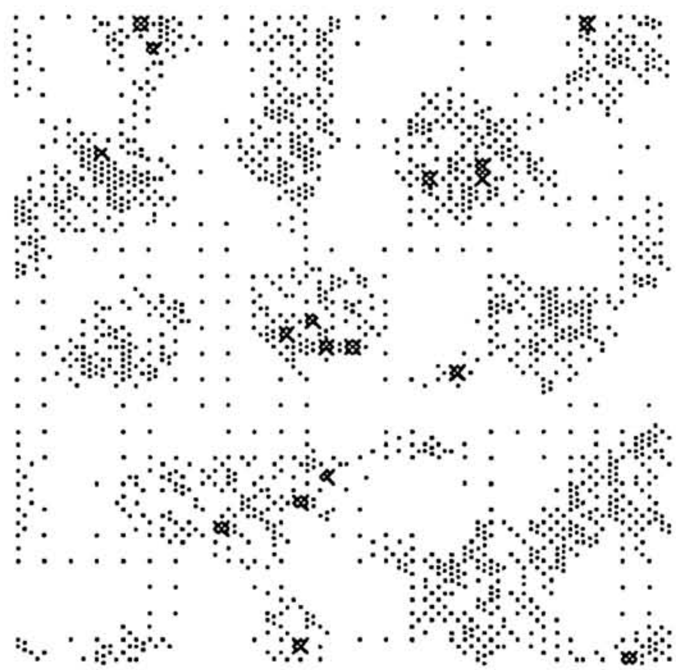

Figure 5. Plot of $|\mathrm{grad}\theta_i|$ corresponding to the orientation map of figure 2. Regions of maximum rate of change of θ_i are shown as shaded. These correspond with the singular regions of figure 2.

Once again, if H_{IO} is taken to be the energy of a gradient dynamical system, there results the equation:

$$\frac{d}{dt}\sigma_i = -\frac{\partial}{\partial\sigma_i}H_{IO} = \sum_{j\neq i} w_{ij}\sigma_j \qquad (7)$$

which is exactly that equation introduced by Swindale in 1981 as a model for the structure of iso-orientation patches. There is an obvious relationship between such equations, and recent similar treatments (Durbin & Mitchison 1990; Schulten, K. 1990 (Preprint); Cherjnavsky & Moody, 1990).

2 CONCLUSIONS

Simple classical spin models well-known to physicists as the ANNNI and Heisenberg XY Models, in which long-range interactions occur in a pattern given by the Mexican Hat operator, can generate many of the structural properties characteristic of the ocular dominance columns and iso-orientation patches seen in cat and primate visual cortex.

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