

1 We thank the reviewers for their detailed comments and insightful suggestions. Below we address some specific
2 comments; we apologize for conciseness due to space constraints. (We thank Reviewers 1 and 8 for supportive reviews!).

3 **Reviewer 3:**

4 We thank the reviewer for a detailed and thorough review. Again, we apologize for our brevity; the nature of this topic
5 warrants a much more careful and elaborate discussion, and it is unfortunate that we cannot do that here.

- 6 1. We were indeed inaccurate here: we will add an explicit projection step to the exposition of SGD and GD. As we
7 stress below, this does not affect our technical development in any crucial way.
- 8 2. As stated, there is indeed a formal projection. Note that in all our constructions the trajectory remains at a bounded
9 constant sized-ball. Thus, for a bounded and closed W , the projection step is avoided and does not affect our
10 constructions—we will highlight this in the proofs where needed.
- 11 3. Indeed, Thm. 1 does not imply that generalization cannot be explained via implicit bias (we do not make such a claim).
12 It only discusses strongly convex regularizers. As such, it definitely does not rule out the possibility of a bias towards
13 solutions with *small enough norm*. Thm. 2, on the other hand, shows that for any (admissible) regularizer we can
14 construct an instance where SGD converges to a point w_* even though there exists another point w_r that has the same
15 empirical error but (strictly) better regularization penalty. We follow here the intuition that if r models the implicit
16 bias of SGD then, given two solutions with same empirical error, SGD needs to choose (approximately) the one with
17 smaller regularization penalty.
- 18 4. When comparing the output of SGD after enough iterations over the unregularized regression loss vs. ridge regression
19 solution with fixed λ : in this case the output of SGD and the output of ridge regression solution are incomparable (in
20 the Pareto-optimality sense). Namely, SGD will have a larger regularization penalty whereas ridge regression will
21 have a larger empirical error; as such, it does not prove Thm. 1. The theorem states the existence of a problem where
22 SGD converges to a solution that is not Pareto-efficient w.r.t. the empirical loss and ℓ_2 norm (not even approximately).
- 23 5. The reviewer is correct here that this requires further explanation. First, note that the assumptions of admissibility are
24 used (and stated) only in Thm. 2, where we rule out *distribution-independent* regularizer. Any regularizer where
25 $r(0) \neq \min r(w)$ can be ruled out in this setting by simply considering the zero function (i.e., $f(w, z) = 0$ for all
26 z). In this case SGD converges to zero, which by assumption is r -suboptimal. Hence, we may assume here that
27 $r(0) = \min r(w)$ and we only need to normalize by choosing $\min r(w) = 0$. (again, we emphasize that this concerns
28 only distribution independent bias; in the distribution dependent section we are not making any assumption on the
29 regularizer to begin with). Following this discussion we will revise the assumption to $\min r(w) = 0$ (and not $r(0) = 0$)
30 and incorporate the discussion above in Thm. 2 where necessary.

31 Several other remarks:

- 32 • We thank the reviewer for indicating some important related work. The suggested lines of work are indeed relevant
33 and should be discussed. Thank you for pointing those out!
- 34 • Two works which we discuss that give evidence for training neural networks without regularizations are [14,22].
35 Specifically, [14] shows how training to zero training error with overcapacitated networks can improve test performance.
- 36 • The remark on where SGD is identical to introducing ℓ_2 regularization: note that this claim is only for *linear*
37 optimization (hence not relevant for regression). Perhaps, a better reference for this fact is Shalev Shwartz “Online
38 Learning and Online Convex Optimization” (see Examples 2.3 therein).

39 **Reviewer 7:**

- 40 • *Typo in the notion of statistically complex set?*: No, the quantifiers are okay. Note that we want to understand if
41 the structure of the set K can explain generalization, and without further considerations of the specific problem at
42 hand. For that one must choose a complexity measure that is independent of D . Note that any (successful) learning
43 algorithm converges, on a specific distribution, to a set of solutions that are okay for that distribution – thus changing
44 the quantifiers will lead to a tautology. What we desire here is to measure the complexity, or understand the structure
45 of the set of solutions w.r.t. any acceptable distribution. Specifically, Thm. 4 shows that on a given instance the set of
46 solution can be “too rich” in the sense that the capacity of the set cannot explain generalization.
- 47 • *Remarks on quantification and improving clarity of Thm. 4:* We will elaborate and clarify here. The quantification
48 over the regularizer is before the sample (i.e., the regularizer may depend on the distribution D but is independent of
49 the sample S). We will also clarify the relationship between d and T ; in a nutshell, the asymptotic variable is T (and
50 $d = O(T)$, $\eta = 1/\sqrt{T}$). Thanks for these suggestions!