
Learning to Solve SMT Formulas

Supplementary Material

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We provide two appendices. Appendix 1 includes in depth descriptions of the algorithms used for learning and synthesis. Appendix 2 provides two additional experiments - evaluating with time limit of 10 minutes and the effect of iterative training used by Algorithm 1.

1 Additional details on learning and synthesis

Finding most likely strategies As described in Algorithm 1, a key step of training is a procedure that finds the top K most likely strategies to solve a given formula (line 4 of Algorithm 1). This algorithm, shown in Algorithm 2, takes as input a single formula $f \in \mathcal{F}$, a model π (e.g., a neural network policy, bilinear model, etc.) and integer K (number of strategies to explore).

During the search we keep a priority queue of tuples (s_j, a_j, p_j) consisting of a state, possible action and its associated probability, initialized with $(s_0, \epsilon, 1)$, s_0 denoting initial state. At each step, we remove the tuple with highest probability from the queue, apply its action to obtain a new state s'_j and update the priority queue with new tuples $(s'_j, a, p_j \cdot \pi(a | s'_j))$ for all actions $a \in \text{Tactics} \times \text{Params}$ capturing the possible transitions from s'_j . For practical reasons we approximate the set of all possible actions (denoted as $\text{ACTIONS}(s'_j, \pi)$) as follows:

- If we are using the neural network policy, we consider the most likely parameters for each tactic according to that policy, or
- If we are not using the neural network policy and instead are using models which do not predict parameters, we consider 20 different parameter configurations for each tactic which are selected at random before training starts.

As described in Section 4.1, we additionally perform pruning of states (line 9) which can not possibly lead to an optimal strategy. Finally, we note that in practice we perform the search for batch of formulas at once in order to leverage parallelization capabilities of our system.

Algorithm 2: Procedure for finding top K strategies

Data: Formula f , Model π , Number of strategies to sample K

Result: Top K most likely strategies according to the model

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1  $s_0 = (f, \epsilon)$ 
2  $S = \emptyset$ 
3  $queue = \text{PRIORITY\_QUEUE}()$ 
4  $queue.PUSH(s_0, \epsilon, 1)$ 
5 for  $j = 1 : K$  do
6    $(s_j, a_j, p_j) = queue.POP()$ 
7    $s_j \xrightarrow{a_j} s'_j$  ▷ State  $s'_j$  is obtained by applying action  $a_j$  in state  $s_j$ 
8    $S \leftarrow S \cup \{s'_j.strategy\}$  ▷ Strategy is extracted from state  $s'_j$  and added to  $S$ 
9   if  $\neg \text{PRUNED}(s'_j)$  then
10    for  $a \in \text{ACTIONS}(s'_j, \pi)$  do
11       $queue.PUSH(s'_j, a, p_j \cdot \pi(a | s'_j))$ 
12 return  $S$ 

```

Building the training dataset In Section 4.1 we described how we construct the dataset used to train the neural network policy. Here we illustrate the process on a concrete example. Let us consider an example where the model explored seven different states as shown in Fig. 4 (left). Further, in addition to the visited states we also keep information about the cumulative runtime required to compute the state (starting from the initial state s_0) as well as whether the state successfully solves the formula. Then, according to the procedure described in Section 4.1, the dataset is constructed by generating one training sample $\langle (\sigma(t_i), \mathbf{p}_i), s_i \rangle$ for each non-terminal state that eventually succeeded in solving the formula. In our example, this corresponds to generating one training sample for states s_1, s_2 and s_3 as shown in Table 4 where we use ϵ to denote that no tactic was applied so far. Note that for state s_1 there are two possible tactics which lead to solving the formula (`simplify(flat = true)` and `bit_blast`) and therefore we assign non-zero target probability to both of these tactics. However, the best strategies in respective subtrees have different runtimes hence probabilities assigned to the corresponding tactics are different as shown in Table 4. Finally, in case both states s_6 and s_7 would not solve the formula, it would mean that no training example is generated for state s_3 (since we generate training samples only for non-terminal states that eventually succeeded in solving the formula).

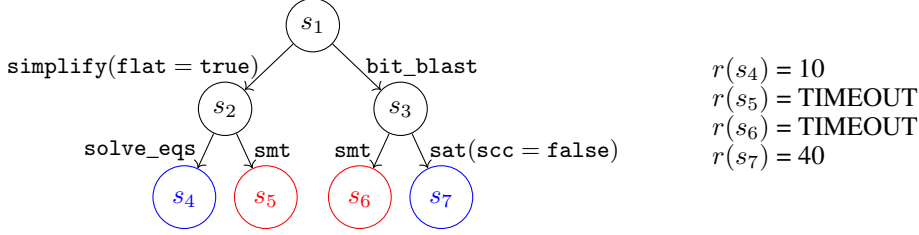


Figure 4: Example with all states visited during solving a formula. Terminal states are colored blue if formula was solved and red otherwise. Runtime for each terminal state is shown on the right.

State	Target tactics	Target parameters
$s_1 = (\varphi_1, \epsilon)$	$\text{Pr}(\text{simplify}) = 0.8$ $\text{Pr}(\text{bit_blast}) = 0.2$	$\text{flat} = \text{true}$ -
$s_2 = (\varphi_2, \text{simplify}(\text{flat} = \text{true}))$	$\text{Pr}(\text{solve_eqs}) = 1$	-
$s_3 = (\varphi_3, \text{bit_blast})$	$\text{Pr}(\text{sat}) = 1$	$\text{scc} = \text{false}$

Table 4: Dataset constructed from the example shown in Fig. 4.

Reducing the set of strategies As the set of synthesized strategies can be large we perform synthesis (described in Section 4.2) only on a subset set of strategies. Intuitively, these strategies should be: strong individually (i.e. each strategy should be able to solve large number of formulas alone) and strong together (i.e. number of formulas solvable by at least one of the strategies should be large). In order to trade-off these two conditions we use greedy procedure shown in Algorithm 3.

We proceed in an iterative manner, choosing one new strategy at every step. In every iteration, strategy receives a score for every formula that it can solve. Score is equal to λ for every formula which was previously unsolved, and $1 - \lambda$ if formula was already solved (by another previously selected strategy). One can notice that if $\lambda = 1$ algorithm will try to greedily maximize the number of formulas that strategies can solve in the union. If $\lambda = 1/2$ algorithm will simply select k strategies which can solve the most formulas individually. In our experiments we treat λ as a hyperparameter and optimize it on a validation set of formulas. Concretely, we set λ to 0.5, 0.99, 0.75, 0.95, 0.7 for Sage2, AProVE, leipzig, core and hycomp respectively.

Algorithm 3: Greedy strategies selection

Data: Set of formulas \mathcal{F} , Strategies s_1, \dots, s_m , weight $\lambda \in [0, 1]$

Result: Set S consisting of k selected strategies

Initialize $S_{best} = \emptyset$

Initialize $A = \mathcal{F}, B = \emptyset$

for $iter = 1 : k$ **do**

for $i = 1 : m$ **do**

if $s_i \notin S_{best}$ **then**

 Calculate $A_i =$ subset of formulas in A which strategy s_i can solve

 Calculate $B_i =$ subset of formulas in B which strategy s_i can solve

 Define $score_i = \lambda|A_i| + (1 - \lambda)|B_i|$

 Add strategy s_i with highest corresponding $score_i$ to S_{best}

$A = A \setminus A_i$

$B = B \cup A_i$

2 Additional experiments

Evaluation with 10 minute time limit The 10 second time limit in our experiments was selected for practical purposes - it is large enough to solve 83.3% of the formulas and to learn strategies in a reasonable time. To check how well our strategies generalize to higher time limits we kept the 10 seconds time limit for training, but used 10 minutes for evaluation. We show results from this experiment in Table 6. With the 10 minute time limit, 88.1% of formulas are solved by both methods. Crucially, our strategies are still able to solve 8.6% more formulas than Z3. In addition, the speedups over Z3 are comparable (and even slightly higher) to speedups achieved with 10 second time limit. For comparison, Table 2 shows the results of evaluating the same strategies on the same set of formulas using a 10 second time limit. Note since the experiments take significantly longer to run we evaluated them only on a subset of the dataset.

Evaluation on larger set of formulas For completeness, we also include the results of the best strategies synthesized by our models on the full set of formulas in our test set using the 10 second time limit. Results are shown in Table 5. The set of formulas is the same as in Table 3 (as opposed to Table 2 and Table 6 which use a subset of the formulas).

Number of basic operations and runtime As stated in Section 4, we use the number of basic operations as a deterministic measure of the amount of work required to solve a formula. In Table 7, we show a comparison between the number of operations and wall clock time for the experiments in Table 6. Note that wall clock time is often imprecise. Especially for formulas which can be solved very fast, wall clock time mostly accounts for initialization of the solver and overhead of the system.

Effect of iterative training In Fig. 5 we show the improvement of our neural network policy as it is continuously retrained using DAgger. We perform a total of 10 stages of DAgger, retraining the model after every stage. In every stage, the current model is used to search for the best strategies, as described in Algorithm 2.

For the purpose of this experiment, we save the models after 2, 4, 6, 8 and 10 stages of DAgger. Then we load each model again and use it to search for the best strategies on the unseen formulas from the test set. We run all of the models for 100 iterations without retraining (which means that each model predicts 100 best strategies for every formula).

One can notice that later models tend to outperform earlier models, thus justifying the increased number of training stages. The only exception in this case are models trained after 6 and 8 stages where an earlier model performs better by a small margin. This can be explained by the stochastic nature of the training procedure.

Tactics For completeness, we also include the set of tactics and parameters used in our experiments. Tactics used for Sage2 and core, both in QF_BV logic are shown in Table 8. Tactics for hycomp in QF_NRA logic are in Table 9. Finally, tactics for leipzig and AProVE in logic QF_NIA are in Table 10. For every tactic we list the parameters that we used (all parameters not listed here are set to the default value) as well as their types.

Table 5: Comparison of best strategy found by any of our models (Section 4.1) against Z3.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	90 th	50 th	10 th
leipzig	57	0	3	8	5.8×	60.7×	191.5×
Sage2	2531	0	2402	1511	1.2×	2.5×	22.0×
core	270	0	0	0	1.2×	1.3×	1.9×
hycomp	1633	1	210	138	1.0×	2.0×	4.3×
AProVE	1397	3	221	91	4.0×	89.6×	988.7×
Total	56.2%	0.1%	27.0%	16.7%	2.6×	31.2×	241.7×

Table 6: Comparison of best strategy found by any of our models against Z3 with 10min time limit.

	Formulas solved				Speedup percentile		
	Both	Only Z3	Only Learned	None	10 th	50 th	90 th
leipzig	63	0	1	4	3.5×	43.9×	183.2×
Sage2	630	0	138	32	1.3×	6.5×	199.6×
core	270	0	0	0	1.2×	1.3×	1.9×
hycomp	298	0	10	17	1.0×	2.0×	40.1×
AProVE	306	0	3	6	3.9×	89.3×	1301.5×
Total	88.1%	0.00%	8.6%	3.3%	2.2×	28.6×	345.3×

Table 7: Comparison of speedup in number of operations and wall clock time.

	Speedup - number of operations				Speedup - Wall clock time			
	P_{10}	P_{50}	P_{90}	Mean	P_{10}	P_{50}	P_{90}	Mean
leipzig	3.5×	43.9×	183.2×	71.6×	0.3×	2.3×	7.3×	3.5×
Sage2	1.3×	6.5×	199.6×	62.7×	1.2×	4.8×	72.5×	37.8×
core	1.2×	1.3×	1.9×	1.4×	0.5×	0.8×	1.3×	0.9×
hycomp	1.0×	2.0×	40.1×	51.9×	0.9×	1.4×	65.7×	25.0×
AProVE	3.9×	89.3×	1301.5×	519.9×	0.9×	6.4×	120.8×	45.8×
Total	2.2×	28.6×	345.3×	141.5×	0.8×	3.1×	53.5×	22.6×

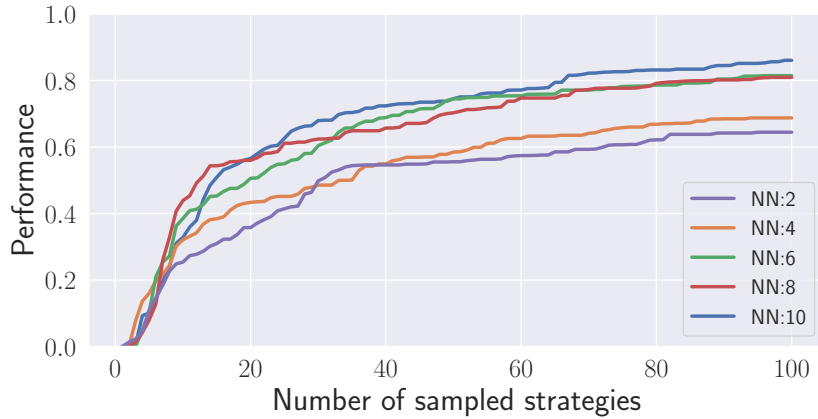


Figure 5: Performance of neural network after 2, 4, 6, 8 and 10 retraining iterations.

Table 8: Tactics and parameters used for QF_BV logic (Sage2 and Core benchmarks).

Tactics	Parameters	Type
simplify	elim_and	bool
	blast_distinct	bool
	push_ite_bv	bool
	som	bool
	pull_cheap_ite	bool
	hoist_mul	bool
	local_ctx	bool
	flat	bool
smt	-	-
bit_blast	-	-
bv1_blast	-	-
solve_eqs	-	-
aig	aig_per_assertion	bool
qfnra_nlsat	-	-
sat	-	-
max_bv_sharing	-	-
reduce_bv_size	-	-
purify_arith	-	-
propagate_values	push_ite_bv	bool
elim_uncnstr	-	-
ackermannize_bv	-	-

Table 9: Tactics and parameters used for QF_NRA logic (hycomp benchmark).

Tactics	Parameters	Type
simplify	elim_and	bool
	blast_distinct	bool
	som	bool
	hi_div0	bool
	hoist_mul	bool
	local_ctx	bool
	flat	bool
smt	-	-
bit_blast	-	-
solve_eqs	-	-
	-	-
qfnra_nlsat	inline_vars	bool
	factor	bool
	seed	int
	-	-
max_bv_sharing	-	-
propagate_values	push_ite_bv	bool
elim_uncnstr	-	-
nla2bv	nla2bv_max_bv_size	int
ctx_simplify	-	-

Table 10: Tactics and parameters used for QF_NIA logic (leipzig and AProVE benchmarks).

Tactics	Parameters	Type
simplify	elim_and	bool
	som	bool
	blast_distinct	bool
	flat	bool
	hi_div0	bool
	local_ctx	bool
	hoist_mul	bool
propagate_values	push_ite_bv	bool
smt	-	-
bit_blast	-	-
solve_eqs	-	-
qfnra_nlsat	-	-
max_bv_sharing	-	-
elim_uncnstr	-	-
nla2bv	nla2bv_max_bv_size	int
ctx_simplify	-	-