

## Supplementary Material

This section presents the complete proofs of lemmas presented in the article.

### A Detailed Proof of Lemma 4.2

**Lemma 4.2** *If there exists a partition in  $S$  such that at least half of its buckets are full, then for the set  $Z$  produced by STAR-T-GREEDY we have*

$$f(Z) \geq (1 - e^{-1}) \left(1 - \frac{4m}{wk}\right) \tau. \quad (2)$$

*Proof.* Let  $i^*$  be a partition such that half of its buckets are full. Let  $B_{i^*,j}$  be a full bucket that minimizes  $|B_{i^*,j} \cap E|$ . In STAR-T, every partition contains  $w \lceil k/2^{i^*} \rceil$  buckets. Hence, the number of full buckets in partition  $i^*$  is at least  $wk/2^{i^*+1}$ . That further implies

$$|B_{i^*,j} \cap E| \leq \frac{2^{i^*+1}m}{wk}. \quad (6)$$

Taking into account that  $B_{i^*,j}$  is a full bucket, we conclude

$$|B_{i^*,j} \setminus E| \geq |B_{i^*,j}| - \frac{2^{i^*+1}m}{wk}. \quad (7)$$

From the property of our Algorithm (line 5) every element added to  $B_{i^*,j}$  increased the utility of this bucket by at least  $\tau/2^{i^*}$ . Combining this with the fact that  $B_{i^*,j}$  is full, we conclude that the gain of every element in this bucket is at least  $\tau/|B_{i^*,j}|$ . Therefore, from Eq. (7) it follows:

$$f(B_{i^*,j} \setminus E) \geq \left(|B_{i^*,j}| - \frac{2^{i^*+1}m}{wk}\right) \frac{\tau}{|B_{i^*,j}|} = \tau \left(1 - \frac{2^{i^*+1}m}{|B_{i^*,j}|wk}\right). \quad (8)$$

Taking into account that  $2^{i^*+1} \leq 4|B_{i^*,j}|$  this further reduces to

$$f(B_{i^*,j} \setminus E) \geq \tau \left(1 - \frac{4m}{wk}\right). \quad (9)$$

Finally,

$$\begin{aligned} f(Z) = f(\text{GREEDY}(k, S \setminus E)) &\geq (1 - e^{-1})f(\text{OPT}(k, S \setminus E)) \\ &\geq (1 - e^{-1})f(\text{OPT}(k, B_{i^*,j} \setminus E)) \end{aligned} \quad (10)$$

$$= (1 - e^{-1})f(B_{i^*,j} \setminus E) \quad (11)$$

$$\geq (1 - e^{-1}) \left(1 - \frac{4m}{wk}\right) \tau, \quad (12)$$

where Eq. (10) follows from  $(B_{i^*,j} \setminus E) \subseteq (S \setminus E)$ , Eq. (11) follows from the fact that  $|B_{i^*,j}| \leq k$ , and Eq. (12) follows from Eq. (9).  $\square$

### B Detailed Proof of Lemma 4.3

We start by studying some properties of  $E$  that we use in the proof of Lemma 4.3.

**Lemma B.1** *Let  $B_i$  be a bucket in partition  $i > 0$ , and let  $E_i := B_i \cap E$  denote the elements that are removed from this bucket. Given a bucket  $B_{i-1}$  from the previous partition such that  $|B_{i-1}| < 2^{i-1}$  (i.e.  $B_{i-1}$  is not fully populated), the loss in the bucket  $B_i$  due to the removals is at most*

$$f(E_i | B_{i-1}) < \frac{\tau}{2^{i-1}} |E_i|.$$

*Proof.* First, we can bound  $f(E_i | B_{i-1})$  as follows

$$f(E_i | B_{i-1}) \leq \sum_{e \in E_i} f(e | B_{i-1}). \quad (13)$$

Consider a single element  $e \in E_i$ . There are two possible cases:  $f(e) < \frac{\tau}{2^{i-1}}$ , and  $f(e) \geq \frac{\tau}{2^{i-1}}$ . In the first case,  $f(e | B_{i-1}) \leq f(e) < \frac{\tau}{2^{i-1}}$ . In the second one, as  $|B_{i-1}| < 2^{i-1}$  we conclude  $f(e | B_{i-1}) < \frac{\tau}{2^{i-1}}$ , as otherwise the streaming algorithm would place  $e$  in  $B_{i-1}$ . These observations together with (13) imply:

$$f(E_i | B_{i-1}) < \sum_{e \in E_i} \frac{\tau}{2^{i-1}} = \frac{\tau}{2^{i-1}} |E_i|.$$

□

**Lemma B.2** *For every partition  $i$ , let  $B_i$  denote a bucket such that  $|B_i| < 2^i$  (i.e. no partition is fully populated), and let  $E_i = B_i \cap E$  denote the elements that are removed from  $B_i$ . The loss in the bucket  $B_{\lceil \log k \rceil}$  due to the removals, given all the remaining elements in the previous buckets, is at most*

$$f\left(E_{\lceil \log k \rceil} \left| \bigcup_{j=0}^{\lceil \log k \rceil - 1} (B_j \setminus E_j)\right.\right) \leq \sum_{j=1}^{\lceil \log k \rceil} \frac{\tau}{2^{j-1}} |E_j|.$$

*Proof.* We proceed by induction. More precisely, we show that for any  $i \geq 1$  the following holds

$$f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) \leq \sum_{j=1}^i \frac{\tau}{2^{j-1}} |E_j|. \quad (14)$$

Once we show that (14) holds, the lemma will follow immediately by setting  $i = \lceil \log k \rceil$ .

**Base case  $i = 1$ .** Since  $B_0$  is not fully populated and the maximum number of elements in the partition  $i = 0$  is 1, it follows that both  $B_0$  and  $E_0$  are empty. Then the term on the left hand side of (14) for  $i = 1$  becomes  $f(E_1)$ . As  $|B_0| < 1$  we can apply Lemma B.1 to obtain

$$f(E_1) = f(E_1 | B_0) \leq |E_1| \frac{\tau}{2^0}.$$

**Inductive step  $i > 1$ .** Now we show that (14) holds for  $i > 1$ , assuming that it holds for  $i - 1$ . First, due to submodularity we have

$$f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right) \geq f\left(E_{i-1} \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right),$$

and, hence, we can write

$$\begin{aligned} f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) &\leq f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) + f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right) - f\left(E_{i-1} \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) \\ &= f\left(E_i \cup \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right) + f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right) - f\left(E_{i-1} \cup \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right). \end{aligned} \quad (15)$$

Due to monotonicity, the first term can be further bounded by

$$f\left(E_i \cup \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right) \leq f\left(E_i \cup B_{i-1} \cup \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right), \quad (16)$$

and for the third term we have

$$f\left(E_{i-1} \cup \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right) = f\left(E_{i-1} \cup B_{i-1} \cup \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right) \geq f\left(B_{i-1} \cup \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right), \quad (17)$$

where to obtain the identity we used that  $E_{i-1} \cup (B_{i-1} \setminus E_{i-1}) = E_{i-1} \cup B_{i-1}$ .

By substituting the obtained bounds (16) and (17) in (15) we obtain:

$$\begin{aligned} f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) &\leq f\left(E_i \left| B_{i-1} \cup \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right) + f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right) \\ &\leq f(E_i | B_{i-1}) + f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right), \end{aligned} \quad (18)$$

where the second inequality follows by submodularity.

Next, Lemma B.1 can be used (as  $|B_{i-1}| < 2^{i-1}$ ) to bound the first term in (18):

$$f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) \leq \frac{\tau}{2^{i-1}} |E_i| + f\left(E_{i-1} \left| \bigcup_{j=0}^{i-2} (B_j \setminus E_j)\right.\right). \quad (19)$$

To conclude the proof, we use the inductive hypothesis that (14) holds for  $i - 1$ , which together with (19) implies

$$f\left(E_i \left| \bigcup_{j=0}^{i-1} (B_j \setminus E_j)\right.\right) \leq \frac{\tau}{2^{i-1}} |E_i| + \sum_{j=1}^{i-1} \frac{\tau}{2^{j-1}} |E_j| = \sum_{j=1}^i \frac{\tau}{2^{j-1}} |E_j|,$$

as desired.  $\square$

**Lemma 4.3** *If there does not exist partition of  $S$  such that at least half of its buckets are full, then for the set  $Z$  produced by STAR-T-GREEDY we have*

$$f(Z) \geq \left(1 - e^{-1/3}\right) \left(f(B_{\lceil \log k \rceil}) - \frac{4m}{wk} \tau\right),$$

where  $B_{\lceil \log k \rceil}$  is a bucket in the last partition which is not fully populated minimizing  $|B_{\lceil \log k \rceil} \cap E|$  and  $|E| \leq m$ .

*Proof.* Let  $B_i$  denote a bucket in partition  $i$  which is not fully populated ( $B_i \leq \min\{2^i, k\}$ ), and for which  $|E_i|$ , where  $E_i = B_i \cap E$ , is of minimum cardinality. Such bucket exists in every partition  $i$  due to the assumption of the lemma that more than a half of the buckets are not fully populated.

First,

$$f\left(\bigcup_{i=0}^{\lceil \log k \rceil} (B_i \setminus E_i)\right) \geq f(B_{\lceil \log k \rceil}) - f\left(E_{\lceil \log k \rceil} \left| \bigcup_{i=0}^{\lceil \log k \rceil - 1} (B_i \setminus E_i)\right.\right) \quad (20)$$

$$\geq f(B_{\lceil \log k \rceil}) - \sum_{i=1}^{\lceil \log k \rceil} \frac{\tau}{2^{i-1}} |E_i|, \quad (21)$$

where Eq. (20) follows from Lemma D.1 by setting  $B = B_{\lceil \log k \rceil}$ ,  $R = E_{\lceil \log k \rceil}$  and  $A = \bigcup_{i=0}^{\lceil \log k \rceil - 1} (B_i \setminus E_i)$ . As we consider buckets that are not fully populated, Lemma B.2 is used to obtain Eq. (21). Next, we bound each term  $\frac{\tau}{2^{i-1}} |E_i|$  in Eq. (21) independently.

From Algorithm 1 we have that partition  $i$  consists of  $w \lceil k/2^i \rceil$  buckets. By the assumption of the lemma, more than half of those are not fully populated. Recall that  $B_i$  is defined to be a bucket of

partition  $i$  which is not fully populated and which minimizes  $|E_i|$ . Let  $\tilde{E}_i$  be the subset of  $E$  that intersects buckets of partition  $i$ . Then,  $|E_i|$  can be bounded as follows:

$$|E_i| \leq \frac{|\tilde{E}_i|}{\frac{w \lceil k/2^i \rceil}{2}} \leq \frac{2^{i+1} |\tilde{E}_i|}{wk}.$$

Hence, the sum on the left hand side of Eq. (21) can be bounded as

$$\sum_{i=1}^{\lceil \log k \rceil} \frac{\tau}{2^{i-1}} |E_i| \leq \sum_{i=1}^{\lceil \log k \rceil} \frac{\tau}{2^{i-1}} \frac{2^{i+1} |\tilde{E}_i|}{wk} = \frac{4}{wk} \tau \sum_{i=1}^{\lceil \log k \rceil} |\tilde{E}_i| \leq \frac{4|E|}{wk} \tau.$$

Putting the last inequality together with Eq. (21) we obtain

$$f \left( \bigcup_{i=0}^{\lceil \log k \rceil} (B_i \setminus E_i) \right) \geq f(B_{\lceil \log k \rceil}) - \frac{4|E|}{wk} \tau.$$

Observe also that

$$\bigcup_{i=0}^{\lceil \log k \rceil} |B_i \setminus E_i| \leq \bigcup_{i=0}^{\lceil \log k \rceil} |B_i| \leq k + \bigcup_{i=0}^{\lceil \log k \rceil} 2^i \leq 3k,$$

which implies

$$f(\text{OPT}(3k, S \setminus E)) \geq f \left( \bigcup_{i=0}^{\lceil \log k \rceil} (B_i \setminus E_i) \right) \geq f(B_{\lceil \log k \rceil}) - \frac{4|E|}{wk} \tau.$$

Finally,

$$\begin{aligned} f(Z) = f(\text{GREEDY}(k, S \setminus E)) &\geq (1 - e^{-1/3}) f(\text{OPT}(3k, S \setminus E)) \\ &\geq (1 - e^{-1/3}) \left( f(B_{\lceil \log k \rceil}) - \frac{4|E|}{wk} \tau \right) \\ &\geq (1 - e^{-1/3}) \left( f(B_{\lceil \log k \rceil}) - \frac{4m}{wk} \tau \right), \end{aligned} \quad (22)$$

as desired.  $\square$

## C Detailed Proof of Lemma 4.4

**Lemma 4.4** *If there does not exist partition of  $S$  such that at least half of its buckets are full, then for the set  $Z$  produced by STAR-T-GREEDY,*

$$f(Z) \geq (1 - e^{-1}) (f(\text{OPT}(k, V \setminus E)) - f(B_{\lceil \log k \rceil}) - \tau),$$

where  $B_{\lceil \log k \rceil}$  is any bucket in the last partition which is not fully populated.

*Proof.* Let  $B_{\lceil \log k \rceil}$  denote a bucket in the last partition which is not fully populated. Such bucket exists due to the assumption of the lemma that more than a half of the buckets are not fully populated.

Let  $X$  and  $Y$  be two sets such that  $Y$  contains all the elements from  $\text{OPT}(k, V \setminus E)$  that are placed in the buckets that precede bucket  $B_{\lceil \log k \rceil}$  in  $S$ , and let  $X := \text{OPT}(k, V \setminus E) \setminus Y$ . In that case, for every  $e \in X$  we have

$$f(e \mid B_{\lceil \log k \rceil}) < \frac{\tau}{k} \quad (23)$$

due to the fact that  $B_{\lceil \log k \rceil}$  is the bucket in the last partition and is not fully populated.

We proceed to bound  $f(Y)$ :

$$f(Y) \geq f(\text{OPT}(k, V \setminus E)) - f(X) \quad (24)$$

$$\geq f(\text{OPT}(k, V \setminus E)) - f(X \mid B_{\lceil \log k \rceil}) - f(B_{\lceil \log k \rceil}) \quad (25)$$

$$\geq f(\text{OPT}(k, V \setminus E)) - f(B_{\lceil \log k \rceil}) - \sum_{e \in X} f(e \mid B_{\lceil \log k \rceil}) \quad (26)$$

$$\geq f(\text{OPT}(k, V \setminus E)) - f(B_{\lceil \log k \rceil}) - \frac{\tau}{k} |X| \quad (27)$$

$$\geq f(\text{OPT}(k, V \setminus E)) - f(B_{\lceil \log k \rceil}) - \tau, \quad (28)$$

where Eq. (24) follows from  $f(\text{OPT}(k, V \setminus E)) = f(X \cup Y)$  and submodularity, Eq (25) and Eq (26) follow from monotonicity and submodularity, respectively. Eq. (27) follows from Eq. (23), and Eq. (28) follows from  $|X| \leq k$ .

Finally, we have:

$$\begin{aligned} f(Z) = f(\text{GREEDY}(k, S \setminus E)) &\geq (1 - e^{-1}) f(\text{OPT}(k, S \setminus E)) \\ &\geq (1 - e^{-1}) f(\text{OPT}(k, Y)) \end{aligned} \quad (29)$$

$$= (1 - e^{-1}) f(Y) \quad (30)$$

$$\geq (1 - e^{-1}) (f(\text{OPT}(k, V \setminus E)) - f(B_{\lceil \log k \rceil}) - \tau), \quad (31)$$

where Eq. (29) follows from  $Y \subseteq (S \setminus E)$ , Eq. (30) follows from  $|Y| \leq k$ , and Eq. (31) follows from Eq. (28).  $\square$

## D Technical Lemma

Here, we outline a technical lemma that is used in the proof of Lemma 4.3

**Lemma D.1** *For any submodular function  $f$  on a ground set  $V$ , and any sets  $A, B, R \subseteq V$ , we have*

$$f(A \cup B) - f(A \cup (B \setminus R)) \leq f(R \mid A).$$

*Proof.* Define  $R_2 := A \cap R$ , and  $R_1 := R \setminus A = R \setminus R_2$ . We have

$$\begin{aligned} f(A \cup B) - f(A \cup (B \setminus R)) &= f(A \cup B) - f((A \cup B) \setminus R_1) \\ &= f(R_1 \mid (A \cup B) \setminus R_1) \\ &\leq f(R_1 \mid (A \setminus R_1)) \end{aligned} \quad (32)$$

$$= f(R_1 \mid A) \quad (33)$$

$$= f(R_1 \cup R_2 \mid A) \quad (34)$$

$$= f(R \mid A),$$

where (32) follows from the submodularity of  $f$ , (33) follows since  $A$  and  $R_1$  are disjoint, and (34) follows since  $R_2 \subseteq A$ .  $\square$

## E Detailed Proof of Theorem 4.5

Setting  $\tau$  in STAR-T assumes that we know the unknown value  $f(\text{OPT}(k, V \setminus E))$ . In this subsection we show how to approximate that value. First,  $f(\text{OPT}(k, V \setminus E))$  can be bounded in the following way:  $\eta \leq f(\text{OPT}(k, V \setminus E)) \leq k\eta$ , where  $\eta$  denotes the largest value of any of the elements of  $V \setminus E$ , i.e.  $\eta = \max_{e \in (V \setminus E)} f(e)$ . In case we are given  $\eta$ , we follow the same approach as in [8] by considering all the  $O(\log_{1+\epsilon} k)$  possible values of  $f(\text{OPT}(k, V \setminus E))$  from the set  $\{(1+\epsilon)^i \mid i \in \mathbb{Z}, \eta \leq (1+\epsilon)^i \leq k\eta\}$ . For each of the thresholds independently and in parallel we then run STAR-T, and hence build  $O(\log_{1+\epsilon} k)$  different summaries. After the stream ends, on each of the summaries we run algorithm STAR-T-GREEDY and report the maximum output over all the runs.

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**Algorithm 3** Parallel Instances of (STAR-T)

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**Input:** Set  $V, k, w \in \mathbb{N}_+, \eta \in \mathbb{R}$

- 1:  $O = \{(1 + \epsilon)^i \mid \eta \leq (1 + \epsilon)^i \leq k\eta\}$
  - 2: Create a set of instances  $\mathcal{I} := \{\text{STAR-T}(V, k, \eta, w) \mid \eta \in O\}$ , and run all the instances in parallel over the stream.
  - 3: Let  $\mathcal{S} = \{\text{the output of instance } I \mid I \in \mathcal{I}\}$ .
  - 4: **return**  $\mathcal{S}$
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**Algorithm 4** Parallel Instances STAR-T- GREEDY

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**Input:** Family of sets  $\mathcal{S}$ , query set  $E$  and  $k$

- 1:  $Z \leftarrow \arg \max_{S \in \mathcal{S}} \text{GREEDY}(k, S \setminus E)$
  - 2: **return**  $Z$
- 

As this approach runs  $O(\log_{1+\epsilon} k)$  copies of our algorithm, it requires  $O(\log_{1+\epsilon} k)$  more memory space than stated in Theorem 4.1. Furthermore, since we are approximating  $f(\text{OPT}(k, V \setminus E))$  as the geometric series with base  $(1 + \epsilon)$ , our final result is an  $(1 + \epsilon)$ -approximation of the value provided in the theorem.

Unfortunately, the value  $\eta$  might also not be known a priori. However,  $\eta$  is some value among the  $m + 1$  largest elements of the stream. This motivates the following idea. At every moment, we keep  $m + 1$  largest elements of the stream. Let  $L$  denote that set (note that  $L$  changes during the course of the stream). Then, for different values of  $\eta$  belonging to the set  $\{f(e) \mid e \in L\}$  we approximate  $f(\text{OPT}(k, V \setminus E))$  as described above. Here we make a minor difference, as also described in [8]. Namely, instead of instantiating all the copies of the algorithm corresponding to  $\eta \leq (1 + \epsilon)^i \leq km$ , we instantiate copies of the algorithm corresponding to the values of  $f(\text{OPT}(k, V \setminus E))$  from the set  $\{(1 + \epsilon)^i \mid i \in \mathbb{Z}, \eta \leq (1 + \epsilon)^i \leq 2k\eta\}$ . We do so as an element  $e$  can belong to an instance of our algorithm even if  $f(\text{OPT}(k, V \setminus E)) = 2kf(e)$ .

Next, let  $e$  be a new element that arrives on the stream. If  $e$  is not among the  $m + 1$  largest elements of the stream seen so far, we do not instantiate any new copy of our algorithm. On the other hand, if  $e$  should replace another element  $e' \in L$  because  $e'$  does not belong to the  $m + 1$  largest elements of the stream anymore, we redefine  $L$  to be  $(L \setminus \{e'\}) \cup \{e\}$ , and update the instances. The instances are updated as follows: we instantiate copies (those that do not exist already) of our algorithm for  $\eta = f(e)$  as described above; and, any instance of our algorithm corresponding to  $\eta = f(e')$ , but not to any other element of  $L$ , we discard.

To bound the space complexity, we start with the following observation – given an element  $e$ , we do not need to add  $e$  to any instance of our algorithm corresponding to  $f(\text{OPT}(k, V \setminus E)) < f(e)$ . This reasoning is justified by the following: if  $e \in E$ , then it does not matter whether we keep  $e$  in our summary or not; if  $e \notin E$ , then  $f(\text{OPT}(k, V \setminus E)) \geq f(e)$ . Therefore, those thresholds that are less than  $f(e)$  are not a good estimate of the optimum solution with respect to  $e$ . To keep the memory space low, we pass an element  $e$  to the instances of our algorithm corresponding to the of  $f(\text{OPT}(k, V \setminus E))$  being in set  $\{(1 + \epsilon)^i \mid i \in \mathbb{Z}, f(e) \leq (1 + \epsilon)^i \leq 2kf(e)\}$ . Notice that, by the structure of our algorithm,  $e$  will not be added to any instance of our algorithm with threshold more than  $2kf(e)$ .

Putting all together we make the following conclusions. At any point during the execution, every element of  $L$  belongs to at most  $O(\log_{1+\epsilon} k)$  instances of our algorithm. Define  $e_{\min} := \arg \min_{e \in L} f(e)$ . Then by the definition, every element  $a \notin L$  kept in the parallel instances of our algorithms is such that  $f(a) \leq f(e_{\min})$ . This further implies that  $a$  also belongs to at most  $O(\log_{1+\epsilon} k)$  instances corresponding to the following set of values  $\{(1 + \epsilon)^i \mid i \in \mathbb{Z}, f(e_{\min}) \leq (1 + \epsilon)^i \leq 2kf(e_{\min})\}$ . Therefore, the total memory usage of the elements of  $L$  is  $O(m \log_{1+\epsilon} k)$ . On the other hand, since all the elements not in  $L$  belong to at most  $O(\log_{1+\epsilon} k)$  different instances of STAR-T, the total memory those elements occupy is  $O((k + m \log k) \log k \log_{1+\epsilon} k)$ . Therefore, the memory complexity of this approach is  $O((k + m \log k) \log k \log_{1+\epsilon} k)$

## F Additional results for the dominating set problem

In Figure 3 we outline further results for the dominating set problem considered in Section 5.1.

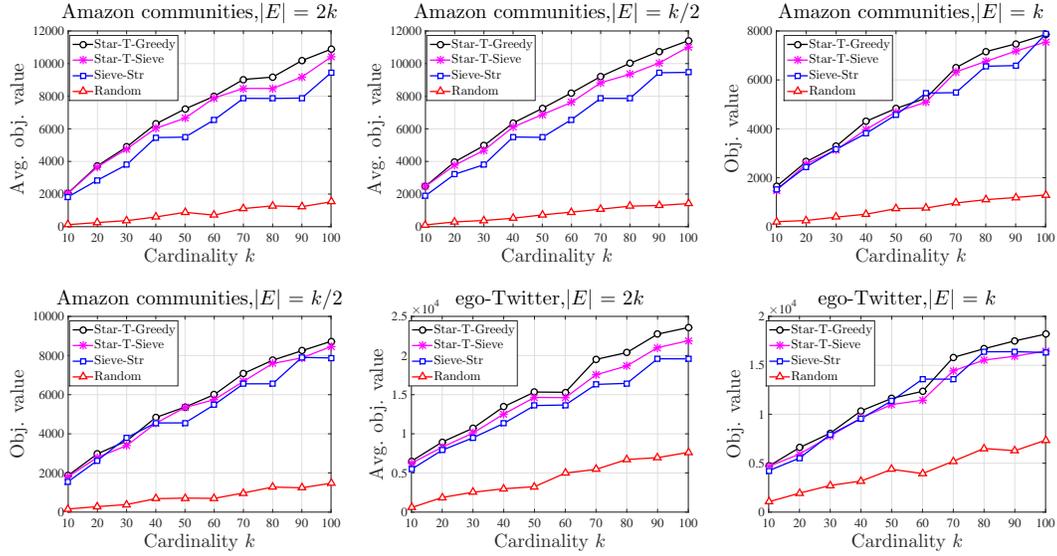


Figure 3: Numerical comparisons of the algorithms STAR-T-GREEDY, STAR-T-SIEVE and SIEVE-STREAMING.