Universal low-rank matrix recovery from Pauli measurements

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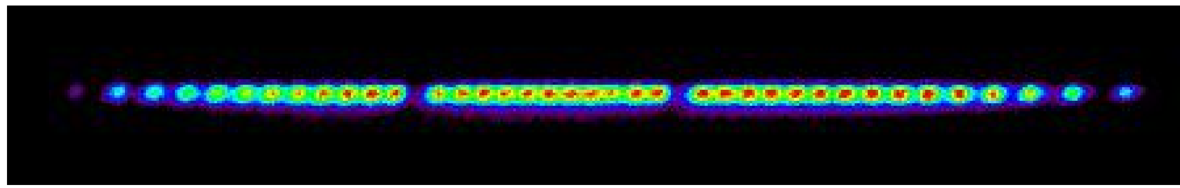
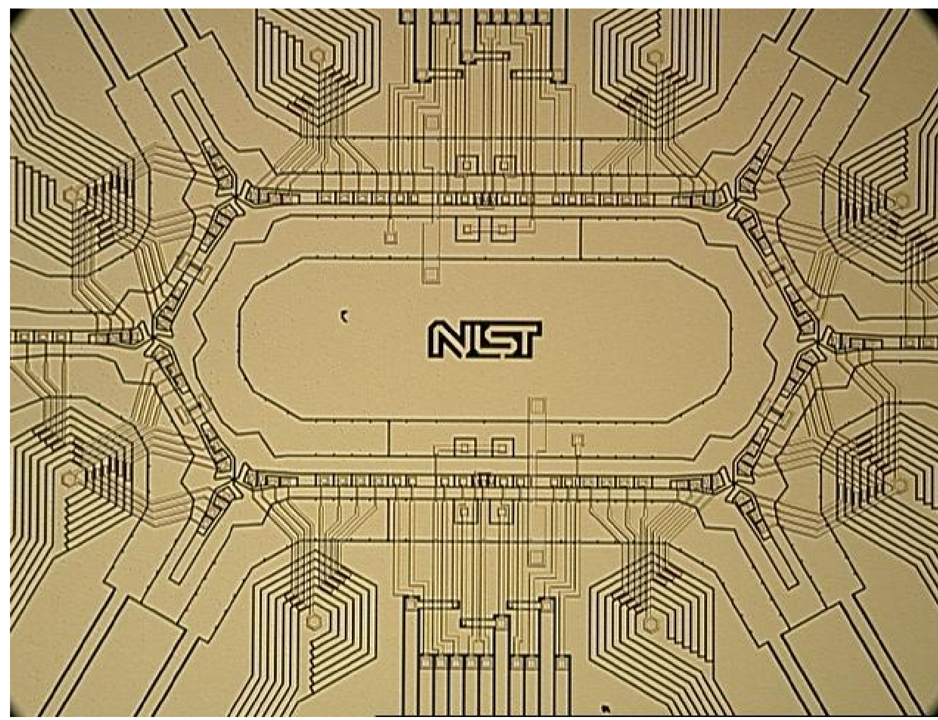
Motivation: experiments with complex quantum systems

- Ion traps





- Small quantum computers
- Precision metrology
- Simulating chemical dynamics
- Want to scale up: 10 to 100 qubits

Quantum state tomography

- Characterizing an unknown quantum state: want to learn the *density matrix* ρ in C^{dxd}
  - For a state of n qubits, $d = 2^n$ => pretty big!
  - In many cases, ρ has rank $r << d$
- Choose measurement matrices P_1, P_2, …
- Observe $Tr(P_1\rho)$, $Tr(P_2\rho)$, …
  - Use Pauli matrices – matrix analogue of Fourier basis
  - **Use compressed sensing techniques!**

Our results

- There is a **universal** set of $O(rd \log^6 d)$ Pauli measurements, that can be used to reconstruct any rank-r state ρ in C^{dxd}
  - Choose random Pauli matrices, use the matrix Lasso
  - Get strong (near-optimal) error bounds [CP'11]

- Random Pauli measurements obey the **restricted isometry property (RIP)**
  - Embed the manifold of low-rank matrices into $O(rd \log^6 d)$ dimensions
  - More structured, less random than a Gaussian random projection