PlasticityNet: Learning to Simulate Metal, Sand, and Snow for Optimization Time Integration: Supplemental Document

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A Appendix

A.1 Network Architecture

All our models are using the Multilayer Perceptron (MLP) architecture with Swiss activation functions ($x\,\mathrm{sigmoid}(x)$) except the output layer. They are trained using ADAM optimizer with the same parameters: initial learning rate $\alpha = 0.01$, decay rate $\gamma = 0.95$, decay step 1000. The dataset is generated during the training process with random sampling, and the batch size is 2^{16} for all cases. The models are all trained with 20000 epochs. The detailed architectures for each model is listed in Table 1.

Table 1: Network Architectures and Training Details

Model	MLP layers
Sand (StVK+Drucker-Prager)	[8,32,32,32,1]
Snow (Neohooken+NACC)	[9,32,32,32,1]
Metal (StVK+von-Mises)	[9,32,32,32,1]
Sand 3D	[18,64,64,64,1]
Metal 3D (StVK+von-Mises)	[19,64,64,64,1]
Snow 3D	[19,64,64,64,1]

A.2 Technical Details on Plasticity Models

We focus on isotropic materials, where the elasticity and plasticity can both be described in the diagonal space without loss of generality. Given the polar singular value decomposition of the deformation gradient $\mathbf{F} = \mathbf{U}\,\mathrm{Diag}(\mathbf{\Sigma})\mathbf{V}^\top$, the Kirchhoff stress $\boldsymbol{\tau}$ and the return mapping $\mathcal{Z}$ can both be computed solely by $\mathbf{\Sigma}$ as $\boldsymbol{\tau}^\Sigma$ and $\mathcal{Z}^\Sigma$, and then restored to the full space via $\boldsymbol{\tau} = \mathbf{U}\,\mathrm{Diag}(\boldsymbol{\tau}^\Sigma)\mathbf{V}^\top$

36th Conference on Neural Information Processing Systems (NeurIPS 2022).

and $\mathcal{Z} = \mathbf{U}\,\mathrm{Diag}(\mathcal{Z}^{\Sigma})\mathbf{V}^{\top}$. In the following sections, we will omit the superscript Σ and only discuss the models in the diagonal space.

Here is a list of the material parameters that will be mentioned in the following sections:

Notation	Meaning	Relation to (E, ν)
E	Young's modulus	/
ν	Poisson's ratio	/
μ	Shear modulus	$\mu = \frac{E}{2(1+\nu)}$
λ	Lamé modulus	$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$
K	Bulk modulus	$K = \frac{E}{3(1-2\nu)}$

A.2.1 Sand Plasticity

We use StVK elasticity and Drucker-Prager plasticity for sand simulations [1].

The Kirchhoff stress $\boldsymbol{\tau}$ of StVK elasticity is defined as

$$\begin{aligned} \boldsymbol{\epsilon} &= \log(\boldsymbol{\Sigma}), \\ \boldsymbol{\tau} &= 2\mu\boldsymbol{\epsilon} + \lambda\,\mathrm{sum}(\boldsymbol{\epsilon})\mathbf{1}. \end{aligned} \tag{1}$$

The elastic region is characterised in the stress space as:

$$\alpha\,\mathrm{sum}(\tau) + \|\,\mathrm{sum}(\tau) - \frac{\mathrm{sum}(\tau)}{d}\mathbf{1}\| \le 0, \tag{2}$$

where $\alpha = \sqrt{\frac{2}{3}}\frac{2\sin\phi_f}{3-\sin\phi_f}$ and ϕ_f is the friction angle. In our sand examples, ϕ_f is set to $\frac{\pi}{6}$. The elastic region in the stress space is shown in Figure 1.

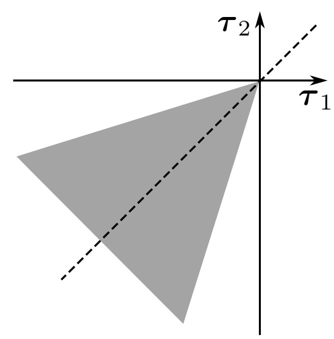

Figure 1: Drucker-Prager plasticity's elastic region in the stress space.

The return mapping for the Drucker-Prager plasticity is

$$\mathcal{Z}(\boldsymbol{\Sigma}) = \begin{cases} \mathbf{1} & \mathrm{sum}(\boldsymbol{\epsilon}) > 0 \\ \boldsymbol{\Sigma} & \delta\gamma \le 0, \text{and}\,\mathrm{sum}(\boldsymbol{\epsilon}) \le 0 \\ \exp\left(\boldsymbol{\epsilon} - \delta\gamma\frac{\hat{\boldsymbol{\epsilon}}}{\|\hat{\boldsymbol{\epsilon}}\|}\right) & \text{otherwise} \end{cases}, \tag{3}$$

where $\delta\gamma = \|\hat{\boldsymbol{\epsilon}}\| + \alpha\frac{(d\lambda+2\mu)\,\mathrm{sum}(\boldsymbol{\epsilon})}{2\mu}$.

A.2.2 Snow Plasticity

We use neo-Hookean elasticity and non-associative Cam-Clay (NACC) plasticity for snow simulations [2].

The Kirchhoff stress of neo-Hookean elasticity is

$$\begin{aligned} J &= \det\mathrm{Diag}(\boldsymbol{\Sigma}), \\ \mathbf{b} &= \boldsymbol{\Sigma}^2, \\ \widehat{\mathbf{b}} &= \mathrm{dev}(\mathbf{b}) = \mathbf{b} - \frac{\mathrm{sum}(\mathbf{b})}{d}\mathbf{1}, \\ \boldsymbol{\tau} &= \mu J^{-\frac{2}{d}}\widehat{\mathbf{b}} + \frac{K}{2}(J^2-1)\mathbf{1}. \end{aligned} \tag{4}$$

The elastic region of NACC is characterized by

$$y(p,q) = q^2(1+2\beta) + M^2(p+\beta p_0)(p-p_0) \le 0, \tag{5}$$

where

$$
\begin{aligned}
M &= d\sqrt{\frac{6-d}{2}}\sqrt{\frac{2}{3}}\frac{2\sin\phi_f}{3-\sin\phi_f} \\
p &= \frac{K}{2}(J^2-1), \\
q &= \mu J^{-\frac{2}{d}}\sqrt{\frac{6-d}{2}}\|\widehat{\mathbf{b}}\|, \\
p_0 &= K\sinh(\xi\max\{-\alpha, 0\}).
\end{aligned} \tag{6}
$$

ξ, β, ϕ_f are the parameters of plasticity and α is the hardening state. The elastic region in the stress space is shown in Figure 2. In our snow examples, $\xi = 0.5$, $\beta = 0.3$ and $\phi_f = \frac{\pi}{4}$.

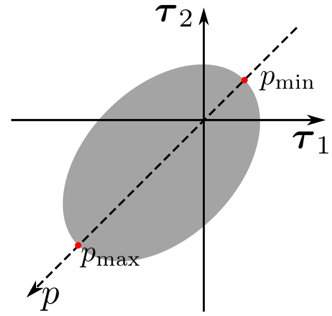

Figure 2: NACC plasticity's elastic region in the stress space.

The return mapping is defined as

$$
\mathcal{Z}(\boldsymbol{\Sigma}) = \begin{cases} (1-\frac{2p_{\text{max}}}{K})^{-\frac{1}{2d}}\mathbf{1}, & p > p_{\text{max}} = p_0, \\ (1+\frac{2p_{\text{min}}}{K})^{-\frac{1}{2d}}\mathbf{1}, & p < p_{\text{min}} = -\beta p_0, \\ \boldsymbol{\Sigma}, & y(p,q) \leq 0, \\ \frac{J^{-\frac{2}{d}}}{\mu}\sqrt{\frac{-2M^2(p+\beta p_0)(p-p_0)}{(6-d)(1+2\beta)}}\frac{\widehat{\mathbf{b}}}{\|\widehat{\mathbf{b}}\|} + \frac{1}{d}\operatorname{sum}(\mathbf{b})\mathbf{1} & \text{Otherwise} \end{cases} \tag{7}
$$

Please refer to [2] for the hardening state update procedure, which is controlled by the simulator. For PlasticityNet, we set $h = \min\{\alpha, 0\}$ as the hardening state input. During the training, we sample $h \in [-0.5, 0]$ for 2D snow and $h \in [-1, 0]$ for 3D snow.

A.2.3 Metal Plasticity under StVK Elasticity

We use StVK elasticity and von-Mises plasticity for metal simulations [1]. This combination provides a closed-form return mapping projection.

The elastic region is characterized by

$$
\|\boldsymbol{\tau} - \frac{1}{d}\operatorname{sum}(\boldsymbol{\tau})\| - \tau_y \leq 0, \tag{8}
$$

where τ_y controls the radius of the yield surface in the stress space (Figure 3).

The return mapping for the von-Mises plasticity is defined as

$$
\mathcal{Z}(\boldsymbol{\Sigma}) = \begin{cases} \boldsymbol{\Sigma}, & \|\boldsymbol{\tau} - \frac{1}{d}\operatorname{sum}(\boldsymbol{\tau})\| - \tau_y \leq 0 \\ \exp\left(\boldsymbol{\epsilon} - \delta\gamma\frac{\hat{\boldsymbol{\epsilon}}}{\|\hat{\boldsymbol{\epsilon}}\|}\right), & \text{Otherwise} \end{cases}, \tag{9}
$$

where $\delta\gamma = \|\hat{\boldsymbol{\epsilon}}\|_F - \frac{\tau_y}{2\mu}$.

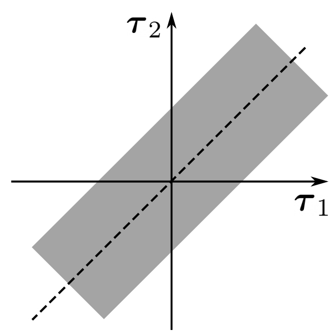

Figure 3: Von-Mises plasticity's elastic region in the stress space.

Under hardening, τ_Y is updated with

$$
\tau_Y^{n+1} = \tau_Y^n + 2\mu\xi\delta\gamma, \tag{10}
$$

where ξ is the hardening coefficient.

We use $h = \frac{\tau_Y}{2\mu}$ as the hardening state input to our PlasticityNet. During the training, h is sampled from $[0, 1]$.

A.2.4 Metal Plasticity under Neo-Hookean Elasticity

The combination of neo-Hookean elasticity and von-Mises plasticity does not have a closed-form return mapping, we thereby use this combination for the task of learning metal plasticity return mapping. The Kirchhoff stress of neo-Hookean elasticity is given by

$$
\boldsymbol{\tau} = \mu(\boldsymbol{\Sigma}^2 - \mathbf{1}) + \lambda\log J\mathbf{1}. \tag{11}
$$

The implicit representation of the elastic region we used in the training of the return mapping is given by

$$y(\mathbf{\Sigma}, h) = \|\boldsymbol{\tau} - \frac{1}{d}\,\mathrm{sum}(\boldsymbol{\tau})\|^2 - (2\mu h)^2. \tag{12}$$

During training, h is sampled from $[0, 1]$.

References

[1] G. Klár, T. Gast, A. Pradhana, C. Fu, C. Schroeder, C. Jiang, and J. Teran. Drucker-prager elastoplasticity for sand animation. *ACM Transactions on Graphics (TOG)*, 35(4):1–12, 2016.

[2] J. Wolper, Y. Fang, M. Li, J. Lu, M. Gao, and C. Jiang. Cd-mpm: continuum damage material point methods for dynamic fracture animation. *ACM Transactions on Graphics (TOG)*, 38(4):1–15, 2019.