Appendix A Action-Conditioned Temporal Abstraction State Transition

We implement action-conditioned state transition

$$p\left(z_t | a_t, z_{<t}, m_{<t}\right) = \begin{cases} \delta(z_t = z_{t-1}) & \text{if } m_{t-1} = 0 \text{ (COPY)}, \\ \tilde{p}(z_t | c_t) & \text{otherwise (UPDATE)} \end{cases}$$

where the action input a_t is only affecting the UPDATE operation and we feed it into the deterministic path as the following:

$$c_t = \begin{cases} c_{t-1} & \text{if } m_{t-1} = 0 \text{ (COPY)}, \\ f_z\left(z_{t-1} \| a_t, c_{t-1}\right) & \text{otherwise (UPDATE).} \end{cases}$$

Appendix B Goal-Oriented Navigation

Algorithm 1 Goal-oriented navigation with imagination (single episode)

Input: environment `env`, environment model $\mathcal{E}$, maximum length of imagined trajectories l_{img}
Output: reward r

Initialize reward $r \leftarrow 100$
Initialize environment $x \leftarrow$ `env.reset`$()$
Initialize context $X_{\text{ctx}} \leftarrow [x]$
Sample goal position and extract goal map feature g
while $r > 0$ **do**
    Sample action a from imagination-based planner given $\mathcal{E}, X_{\text{ctx}}, g, l_{\text{img}}$ (Algorithm 2)
    Do action $x \leftarrow$ `env.step`(a)
    Update context $X_{\text{ctx}} \leftarrow X_{\text{ctx}} + [x]$
    Update reward $r \leftarrow r - 1$
    if current position is at the goal position **then**
        break
return reward r

Algorithm 2 Imagination-based planner

Input: environment model $\mathcal{E}$, previously observed sequence (context) X_{ctx}, maximum length of imagined trajectories l_{img}, goal map feature g
Output: action a_{max}

Initialize model $\mathcal{E}$ with $c_0 = f_{\text{ctx}}\left(X_{\text{ctx}}\right)$
Initialize $d_{\text{max}} \leftarrow -\infty$
Initialize $a_{\text{max}} \leftarrow$ None
Set a list $\mathcal{A}$ of all possible action sequences based on l_{img}
for each action seuqence A in $\mathcal{A}$ **do**
    Get a sequence of states $\mathcal{S} \in \mathbb{R}^{l_{\text{img}} \times d}$ by doing imagination with model $\mathcal{E}$ conditioned on A
    Compute cosine-similarity $D \in \mathbb{R}^{l_{\text{img}}}$ between all states $\mathcal{S}$ and goal map feature g
    if $\max(D) > d_{\text{max}}$ **then**
        Update $d_{\text{max}} \leftarrow \max(D)$
        Update $a_{\text{max}} \leftarrow A[0]$
return action a_{max}

Appendix C Implementation Details

For bouncing balls, we define the reconstruction loss (data likelihood) by using binary cross-entropy. The images from 3D maze are pre-processed by reducing the bit depth to 5 bits (Kingma & Dhariwal, 2018) and therefore the reconstruction loss is computed by using Gaussian distribution. We used the AMSGrad (Reddi et al., 2018), a variant of Adam, optimizer with learning rate $5e-4$ and all mini-batchs are with 64 sequences with length $T = 20$. Both CNN-based encoder and decoder are composed of 4 convolution layers with ELU activations (Clevert et al., 2016). A GRU (Cho et al., 2014) is used for all RNNs with 128 hidden units. The state representations of temporal abstraction and observation abstraction are sampled from 8-dimensional diagonal Gaussian distributions.

Appendix D Evidence Lower Bound (ELBO)

We derive the ELBO without considering recurrent deterministic paths.

Log-likelihood $\log p\left(X\right)$

$$\begin{aligned}
\log p\left(X\right) &= \log \sum_{M} \int_{Z,S} p\left(X, Z, S, M\right) \\
&\geq \sum_{M} \int_{Z,S} q\left(Z, S, M|X\right) \log \frac{p\left(X, Z, S, M\right)}{q\left(Z, S, M|X\right)} \\
&= \sum_{M} \int_{Z,S} q\left(Z, S, M|X\right) \log \frac{p\left(X|Z, S\right) p\left(Z, S, M\right)}{q\left(Z, S, M|X\right)} \\
&= \sum_{M} \int_{Z,S} q\left(Z, S, M|X\right) \left[\log p\left(X|Z, S\right) + \log \frac{p\left(Z, S, M\right)}{q\left(Z, S, M|X\right)}\right] \\
&= \underbrace{\mathbb{E}_{q(Z,S,M|X)}\left[\log p\left(X|Z, S\right)\right]}_{\text{reconstruction}} - \underbrace{\mathrm{KL}\left[q\left(Z, S, M|X\right) || p\left(Z, S, M\right)\right]}_{\text{KL divergence}}
\end{aligned}$$

Decomposing $p\left(X|Z, S\right)$ **and** $p\left(Z, S, M\right)$

$$\begin{aligned}
p\left(X|Z, S\right) &= \prod_{t} \underbrace{p\left(x_t|s_t\right)}_{\text{decoder}} \\
p\left(Z, S, M\right) &= \prod_{t} \underbrace{p\left(z_t|z_{t-1}, m_{t-1}\right)}_{\text{temporal abstract transition}} \underbrace{p\left(s_t|s_{t-1}, z_t, m_{t-1}\right)}_{\text{observation abstract transition}} \underbrace{p\left(m_t|s_t\right)}_{\text{boundary prior}}
\end{aligned}$$

Decomposing $q\left(Z, S, M|X\right)$

$$\begin{aligned}
q\left(Z, S, M|X\right) &= q\left(M|X\right) q\left(Z, S|M, X\right) \\
&= q\left(M|X\right) q\left(Z|M, X\right) q\left(S|Z, M, X\right) \\
&= q\left(M|X\right) \prod_{t} q\left(z_t|M, X\right) q\left(s_t|z_t, M, X\right) \\
q(M|X) &= \prod_{t} q(m_t|X) = \prod_{t} \text{Bern}\left(m_t|\sigma(\varphi(X))\right)
\end{aligned}$$

Reconstruction Term in ELBO

$$\begin{aligned}
&\sum_{M} \int_{Z,S} q\left(Z, S, M|X\right) \log p\left(X|Z, S\right) \\
&= \underbrace{\sum_{M} q\left(M|X\right)}_{\text{sample } M} \int_{Z,S} q\left(Z, S|M, X\right) \log p\left(X|Z, S\right) \\
&\approx \int_{Z,S} q\left(Z, S|M, X\right) \sum_{t} \log p\left(x_t|s_t\right) \\
&= \int_{Z,S} \sum_{t} q\left(z_t, s_t|M, X\right) \log p\left(x_t|s_t\right) \\
&= \int_{Z,S} \sum_{t} \underbrace{q\left(z_t|M, X\right) q\left(s_t|z_t, M, X\right)}_{\text{sampling } z_t \text{ and } s_t} \log p\left(x_t|z_t, s_t\right) \\
&\approx \sum_{t} \log p\left(x_t|s_t\right)
\end{aligned}$$

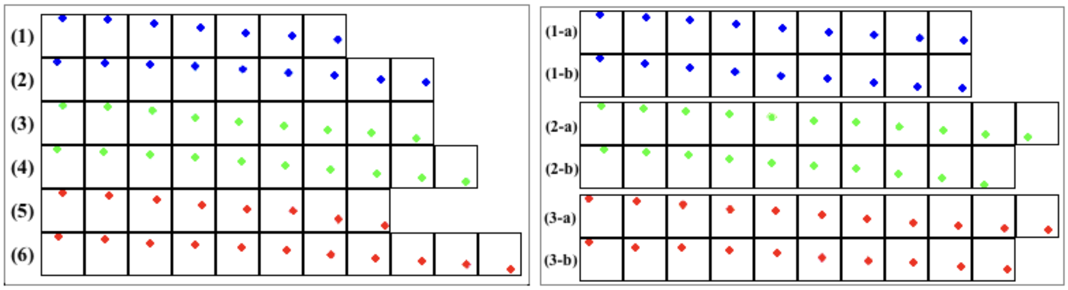
KL Term in ELBO

$$\begin{aligned}
&\log q\left(Z,S,M|X\right)-\log p\left(Z,S,M\right)\\
&=\log q\left(M|X\right)+\log q\left(Z,S|M,X\right)-\log p\left(Z,S,M\right)\\
&=\log q\left(M|X\right)+\log q\left(Z|M,X\right)+\log q\left(S|Z,M,X\right)-\log p\left(Z,S,M\right)\\
&=\log q\left(M|X\right)+\sum_t \log\frac{q\left(z_t|M,X\right)q\left(s_t|z_t,M,X\right)}{p\left(z_t|z_{t-1},m_{t-1}\right)p\left(s_t|s_{t-1},z_t,m_{t-1}\right)p\left(m_t|s_t\right)}\\
&=\log q\left(M|X\right)+\sum_t \log\frac{q\left(z_t|M,X\right)}{p\left(z_t|z_{t-1},m_{t-1}\right)}+\log\frac{q\left(s_t|z_t,M,X\right)}{p\left(s_t|s_{t-1},z_t,m_{t-1}\right)}+\log\frac{1}{p\left(m_t|s_t\right)}
\end{aligned}$$

$$\begin{aligned}
&\sum_M\int_{Z,S} q\left(Z,S,M|X\right)\left[\log q\left(Z,S,M|X\right)-\log p\left(Z,S,M\right)\right]\\
&=\sum_M q\left(M|X\right)\int_{Z,S} q\left(Z,S|M,X\right)\left[\log q\left(Z,S,M|X\right)-\log p\left(Z,S,M\right)\right]\\
&=\sum_M q\left(M|X\right)\int_{Z,S} q\left(Z,S|M,X\right)\left[\log q\left(M|X\right)+\sum_t\log\frac{q\left(z_t|M,X\right)q\left(s_t|z_t,M,X\right)}{p\left(z_t|z_{t-1},m_{t-1}\right)p\left(s_t|s_{t-1},z_t,m_{t-1}\right)p\left(m_t|s_t\right)}\right]\\
&=\sum_M q\left(M|X\right)\left[\log q\left(M|X\right)+\sum_t\int_{Z,S} q\left(Z,S|M,X\right)\log\frac{q\left(z_t|M,X\right)q\left(s_t|z_t,M,X\right)}{p\left(z_t|z_{t-1},m_{t-1}\right)p\left(s_t|s_{t-1},z_t,m_{t-1}\right)p\left(m_t|s_t\right)}\right]\\
&=\sum_M q\left(M|X\right)\left[\log q\left(M|X\right)+\sum_t\int_{z_t,s_t} q\left(z_t,s_t|M,X\right)\log\frac{q\left(z_t|M,X\right)q\left(s_t|z_t,M,X\right)}{p\left(z_t|z_{t-1},m_{t-1}\right)p\left(s_t|s_{t-1},z_t,m_{t-1}\right)p\left(m_t|s_t\right)}\right]\\
&=\sum_M q\left(M|X\right)\left[\log q\left(M|X\right)+\sum_t \mathrm{KL}\left(q'(z_t)||p'(z_t)\right)+\int_{z_t,s_t} q\left(z_t,s_t|M,X\right)\log\frac{q\left(s_t|z_t,M,X\right)}{p\left(s_t|s_{t-1},z_t,m_{t-1}\right)p\left(m_t|s_t\right)}\right]\\
&=\sum_M q\left(M|X\right)\left[\log q\left(M|X\right)+\sum_t \mathrm{KL}\left(q'(z_t)||p'(z_t)\right)+\int_{z_t}\underbrace{q'\left(z_t\right)}_{\text{sample } z_t}\left[\mathrm{KL}\left(q'(s_t)||p'(s_t)\right)-\log p\left(m_t|s_t\right)\right]\right]\\
&\approx\sum_M q\left(M|X\right)\left[\log q\left(M|X\right)+\sum_t \mathrm{KL}\left(q'(z_t)||p'(z_t)\right)+\mathrm{KL}\left(q'(s_t)||p'(s_t)\right)-\log p\left(m_t|s_t\right)\right]\\
&=\sum_M q\left(M|X\right)\left[\log\frac{\prod_t q\left(m_t|X\right)}{\prod_t p\left(m_t|s_t\right)}+\sum_t \mathrm{KL}\left(q'(z_t)||p'(z_t)\right)+\mathrm{KL}\left(q'(s_t)||p'(s_t)\right)\right]\\
&=\sum_{t'}\mathrm{KL}\left(q'\left(m_{t'}\right)||p'\left(m_{t'}\right)\right)+\sum_M\underbrace{q\left(M|X\right)}_{\text{sample } M}\left[\sum_t \mathrm{KL}\left(q'\left(z_t\right)||p'\left(z_t\right)\right)+\mathrm{KL}\left(q'\left(s_t\right)||p'\left(s_t\right)\right)\right]\\
&\approx\sum_t\underbrace{\mathrm{KL}\left(q'\left(m_t\right)||p'\left(m_t\right)\right)}_{\text{sequence decomposer}}+\underbrace{\mathrm{KL}\left(q'\left(z_t\right)||p'\left(z_t\right)\right)}_{\text{temporal abstraction}}+\underbrace{\mathrm{KL}\left(q'\left(s_t\right)||p'\left(s_t\right)\right)}_{\text{observation abstraction}}
\end{aligned}$$

where $p'(m_t)=p(m_t|s_t), p'(z_t)=p\left(z_t|z_{t-1},m_{t-1}\right), p'(s_t)=p\left(s_t|s_{t-1},z_t,m_{t-1}\right)$
$q'(m_t)=q(m_t|X), q'(z_t)=q\left(z_t,s_t|M,X\right), q'(s_t)=q\left(s_t|z_t,M,X\right)$

Appendix E Generated Samples



(a) Subsequences with different z (and different s)

(b) Subsequences with same z (and different s)

Figure 1: Generated subsequences with different content and temporal structure

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