SUPPLEMENTARY MATERIAL
Exact inference in structured prediction

A Detailed Proofs

In this section, we state the proofs of Theorem 3 and Corollaries 1, 2, 3 from our manuscript.

A.1 Proof of Theorem 3

Proof. We are interested in upper bounding the probability of predicting the wrong vector $\boldsymbol{y}$, that is,

$$\begin{aligned} P(\boldsymbol{c}^\top \boldsymbol{y}^* \leq -\boldsymbol{c}^\top \boldsymbol{y}^*) &= P(\boldsymbol{c}^\top \boldsymbol{y}^* \leq 0) \\ &= P\big(\sum_{u \in \mathcal{V}} z_q^{(u)} \leq 0\big) \\ &\leq e^{-\frac{n}{2}(1-2q)^2}, \end{aligned}$$

where for the last equation we applied Hoeffding's inequality. □

A.2 Proof of Corollary 1

Proof. Fix $\varepsilon = \log^8 n$. Let $\epsilon_r(n, \varepsilon) = n^{-2.2 - \frac{\log \varepsilon}{2}}$, then from Lemma 3 we get $\phi_{\widetilde{\mathcal{G}}} \in \Omega(\log^7 n)$ with probability at least $1 - \epsilon_r(n, \varepsilon)$. Let $\Delta_{\max}$ be the maximum node degree of graph $\mathcal{G}$, then it is clear that $\Delta^{\widetilde{\mathcal{G}}}_{\max}$ is a random variable with expected value $\mathbb{E}[\Delta^{\widetilde{\mathcal{G}}}_{\max}] \leq \Delta_{\max} + \log^8 n$. By applying Markov's inequality we obtain $P(\Delta^{\widetilde{\mathcal{G}}}_{\max} \geq t) \leq \mathbb{E}[\Delta^{\widetilde{\mathcal{G}}}_{\max}]/t \leq (\Delta_{\max} + \log^8 n)/t$ for $t > 0$. Set $t = \log^9 n$, then let $\epsilon_\Delta(\Delta_{\max}, n) = (\Delta_{\max} + \log^8 n)/\log^9 n$, we have that $\Delta^{\widetilde{\mathcal{G}}}_{\max} \leq \log^9 n$ with probability at least $1 - \epsilon_\Delta(\Delta_{\max}, n)$.

By using the union bound and noting that $\epsilon_r \to 0$ and $\epsilon_\Delta \to 0$ as $n \to \infty$, we have that $\phi^2_{\widetilde{\mathcal{G}}}/\Delta^{\widetilde{\mathcal{G}}}_{\max} \in \Omega(\log^5 n)$ and $\Delta^{\widetilde{\mathcal{G}}}_{\max} \in \mathcal{O}(\log^9 n)$ with high probability. Finally, this leads to $\epsilon_1 \to 0$ as $n \to \infty$, thus, exact inference is achievable in polynomial time. □

A.3 Proof of Corollary 2

Proof. For any set $\mathcal{S} \subset \mathcal{V}$ with $|\mathcal{S}| \leq n/2$, we have that:

$$\phi_{\mathcal{S}} = \frac{|\mathcal{E}(\mathcal{S}, \mathcal{S}^C)|}{|\mathcal{S}|} = \frac{|\mathcal{S}| \cdot |\mathcal{S}^C|}{|\mathcal{S}|} = |\mathcal{S}^C| \quad \Longrightarrow \quad \phi_{\mathcal{G}} = \lceil \frac{n}{2} \rceil.$$

Since $\mathcal{G}$ is a complete graph, we have that $\Delta_{\max} = n - 1$, which yields $\phi^2_{\mathcal{G}}/\Delta_{\max} \in \Omega(n)$. Thus, from Theorem 2, we have that $\epsilon_1(\phi_{\mathcal{G}}, \Delta_{\max}, p) \to 0$ as $n \to \infty$. □

A.4 Proof of Corollary 3

Proof. From Definition 6, we have that $\phi_{\mathcal{G}} \geq c \cdot d$. Since the graph is regular, we have that $\Delta_{\max} = d$. Therefore, $\phi^2_{\mathcal{G}}/\Delta_{\max} \in \Omega(d)$. Finally, if $d \in \Omega(\log n)$, then $\epsilon_1(\phi_{\mathcal{G}}, \Delta_{\max}, p)$ decays in at least n^{-c_1} for some constant $c_1 > 0$. That is, $\epsilon_1(\phi_{\mathcal{G}}, \Delta_{\max}, p) \to 0$ as $n \to \infty$. □