A Appendix

A.1 Proof of Theorem 3.1

Proof. Since $P_{\text{k-ReLU},i}$ contains $\mathcal{P}_{R,i} \neq \emptyset$ and both $P_{\text{k-ReLU},i}$, $\bigsqcap_{i=1}^{k} P_{\text{1-ReLU},\mathcal{I}_i}$ contain the same interval constraints, we have that $P_{\text{k-ReLU},i} \subsetneqq \bigsqcap_{i=1}^{k} P_{\text{1-ReLU},\mathcal{I}_i}$. We have by monotonicity of the intersection and the convex hull,

$$(\bigsqcup_{S_i \in \mathcal{S}_i} P_{\text{k-ReLU},i} \sqcap S_i) \subsetneqq (\bigsqcup_{S_i \in \mathcal{S}_i} (\bigsqcap_{u \in \mathcal{I}_i} P_{\text{1-ReLU},u}) \sqcap S_i) \tag{9}$$

For any $S_i \in \mathcal{S}_i$, we have that either $S_i \subseteq \mathcal{C}_u^+$ or $S_i \subseteq \mathcal{C}_u^-$ for any $u \in \mathcal{I}_i$. Thus, we can replace all S_i on the right hand side of (9) with either $\mathcal{C}_u^+$ or $\mathcal{C}_u^-$ such that for all $u \in \mathcal{I}_i$ both $\mathcal{C}_u^+$ and $\mathcal{C}_u^-$ are used in at least one substitution and obtain by monotonicity,

$$\begin{aligned}
&\subseteq (\bigsqcup_{j=1}^{k}((\bigsqcap_{u \in \mathcal{I}_i} P_{\text{1-ReLU},u}) \sqcap \mathcal{C}_u^+) \sqcup ((\bigsqcap_{u \in \mathcal{I}_i} P_{\text{1-ReLU},u}) \sqcap \mathcal{C}_u^-)) \\
&\subseteq (\bigsqcup_{u \in \mathcal{I}_i} (P_{\text{1-ReLU},u} \sqcap \mathcal{C}_u^+) \sqcup (P_{\text{1-ReLU},u} \sqcap \mathcal{C}_u^-)).
\end{aligned}$$

For other i's, it can be shown similarly that $(\bigsqcup_{S_i \in \mathcal{S}_i} P_{\text{k-ReLU},i} \sqcap S_i) \subseteq (\bigsqcup_{u \in \mathcal{I}_i} (P_{\text{1-ReLU},u} \sqcap \mathcal{C}_u^+) \sqcup (P_{\text{1-ReLU},u} \sqcap \mathcal{C}_u^-))$ holds. Since $\subsetneqq$ relation holds for at least one i and $\subseteq$ holds for other i's, $P_{\text{k-ReLU}} \subsetneqq P_{\text{1-ReLU}}$ holds by the order preserving property of the intersection. □

A.2 1-ReLU vs 2-ReLU vs 3-ReLU on the 9×200 network

We graphically show the precision and the average runtime of 1-ReLU, 2-ReLU, and 3-ReLU with no MILP and with one layer of MILP on the 9×200 MNIST network with $\epsilon = 0.015$ using the results from the evaluation section in Fig. 4.

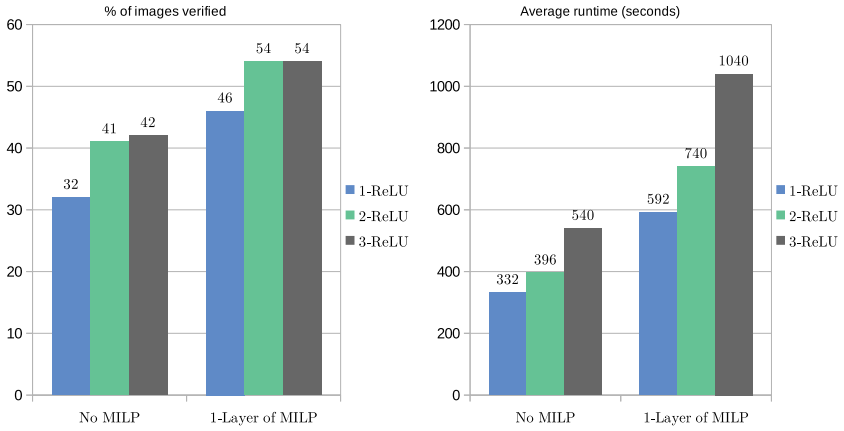

Figure 4: Percentage of images verified and the average runtime (in seconds) of 1-ReLU, 2-ReLU, and 3-ReLU on the 9×200 MNIST network with $\epsilon = 0.015$.