Non-Local Recurrent Network for Image Restoration Supplementary Material

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1 Overview

In this supplementary document, we present additional results to complement the paper. First, we present an extension of our general framework to other classic non-local methods for image restoration. Second, we provide visual results for the comparison of our NLRN and several competing methods on image denoising and image super-resolution.

2 Extension of the General Framework to Other Classic Non-Local Methods

Besides the extension to WNNM and non-local means, which are discussed in Section 3.2 of the main paper, we show the proposed non-local framework can be extended to collaborative filtering methods, *e.g.*, BM3D algorithm [1], as well as joint sparsity based methods, *e.g.*, LSSC algorithm [6]. We follow the same notations in Section 3.2 of the main paper. Both BM3D and LSSC apply block matching (BM) first before processing, and form N groups of similar patches into data matrices. The index set of the matched patches for the *i*-th reference patch is denoted as \mathbb{C}_i . The group of matched patches for the *i*-th reference patch is denoted as $X_{\mathbb{C}_i}$.

Similar to WNNM [3], BM3D [1] also applies BM first to group similar patches based on their Euclidean distances. The matched patches are then processed via Wiener filtering [1], and the denoised results of the *i*-th group of patches are

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$$\mathbf{Z}_{\mathbb{C}_i} = \tau^{-1}(\operatorname{diag}(\omega)\tau(\mathbf{X}_{\mathbb{C}_i})).$$
(1)

Here $\tau(\cdot)$ and $\tau^{-1}(\cdot)$ denote the forward and backward Wiener filtering applied to the groups of matched patches, respectively. The diagonal matrix $\operatorname{diag}(\omega)$ is formed by the empirical Wiener coefficients ω . BM3D applies data pre-cleaning, using discrete cosine transform (DCT), to estimate the original patch, and calculate the estimate of ω [1]. Since calculating $Z_{\mathbb{C}_i}$ in (1) involves only linear filtering, it can also be generalized using the proposed non-local framework. Unlike the extension to WNNM, here $\sum_{j \in \mathbb{C}_i} \Phi(X)_i^j G(X)_j$ corresponds to the denoised results via Wiener filtering as shown in (1), of the *i*-th group of matched patches.

Different from BM3D and WNNM, LSSC learns a common dictionary D for all image patches, and imposes joint sparsity [6] on each data matrix of matched patches $X_{\mathbb{C}_i}$, so that the correlation of the matched patches are exploited by enforcing the same support of their sparse codes. Thus, the joint sparse coding in LSSC [6] becomes

$$\hat{\boldsymbol{A}}_{i} = \operatorname{argmin}_{\boldsymbol{A}_{i}} \left\| \boldsymbol{A}_{i} \right\|_{0,\infty} \quad s.t. \quad \left\| \boldsymbol{X}_{\mathbb{C}_{i}}^{T} - \boldsymbol{D}\boldsymbol{A}_{i} \right\|_{F}^{2} \leq \epsilon \left| \mathbb{C}_{i} \right|, \quad \forall i,$$
(2)

where the $(0, \infty)$ "norm" $\|\cdot\|_{0,\infty}$ counts the number of non-zero columns of each sparse code matrix A_i [6], and $|\mathbb{C}_i|$ is the cardinality of \mathbb{C}_i . The coefficient ϵ is a constant, which is used to upper bound the sparse modeling errors. In general, the solution to (2) is NP-hard. To simplify the discussion, we assume the dictionary to be unitary (which reduces the sparse coding problem to the transform-model

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sparse coding [10]), *i.e.*, $\mathbf{D}^T \mathbf{D} = \mathbf{I}$ and $\mathbf{D} \in \mathbb{R}^{k \times k}$. Thus there exists a corresponding shrinkage function $\eta(\cdot)$ for imposing joint sparsity on the sparse codes [6, 7], such that the denoised estimates of the *i*-th patch group can be obtained as $\mathbf{Z}_{\mathbb{C}_i} = \hat{\mathbf{A}}_i^T \mathbf{D}^T = \eta(\mathbf{X}_{\mathbb{C}_i} \mathbf{D}) \mathbf{D}^T$. Though joint sparse coding projects all data onto a union of subspaces [6, 2, 10] which is a non-linear operation in general, each data matrix $\mathbf{X}_{\mathbb{C}_i}$ is projected onto one particular subspace spanned by the selected atoms corresponding to the non-zero columns in $\hat{\mathbf{A}}_i$, which is locally linear. For the *i*-th group of patches, such a subspace projection corresponds to $\sum_{j \in \mathbb{C}_i} \Phi(\mathbf{X})_i^j \mathbf{G}(\mathbf{X})_j$ in the proposed general framework.

3 Visual Results

We show the visual comparison of our NLRN and several competing methods: BM3D [1], WNNM [3], and MemNet [9] for image denoising in Figure 1. Our method can recover more details from the noisy measurement. The visual comparison of our NLRN and several recent methods: DRCN [4], LapSRN [5], DRRN [8], and MemNet [9] for image super-resolution is displayed in Figure 2. Our method is able to reconstruct sharper edges and produce fewer artifacts especially in the regions of repetitive patterns.



Figure 1: Qualitative comparison of image denoising results with noise level of 30. The zoom-in region in the red bounding box is shown on the right. From top to bottom: 1) the image *barbara*. 2) image *004* in Urban100. 3) image *019* in Urban100. 4) image *033* in Urban100. 5) image *046* in Urban100.



Figure 2: Qualitative comparison of image super-resolution results with $\times 4$ upscaling. The zoom-in region in the red bounding box is shown on the right. From top to bottom: 1) image 005 in Urban100. 2) image 019 in Urban100. 3) image 044 in Urban100. 4) image 062 in Urban100. 5) image 099 in Urban100.

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