A Additional proofs

Lemma 2. *For each iteration k, on the event $\delta_k \le \delta_k^-$, we have*

$$\mathbb{P}(\ell_k' \ge \eta m_k' \mid \mathcal{M}_k^-) \ge \zeta_0\zeta_1 > \frac{1}{2}. \tag{26}$$

Proof. Let A_k denote the event that

$$\mathcal{L}_k' - \eta m_k' \ge \nu_1 \|\nabla \mathcal{L}_k\| \delta_k. \tag{27}$$

By Lemma 3,

$$\begin{aligned} \mathbb{P}(A_k \mid \mathcal{M}_k^-) &\ge \mathbb{P}(c_1\delta_k - c_2\delta_k^2 \ge \nu_1\|\nabla\mathcal{L}_k\|\delta_k \mid \mathcal{M}_k^-) && (28)\\ &= \mathbb{P}(c_1 - \nu_1\|\nabla\mathcal{L}_k\| - c_2\delta_k \ge 0 \mid \mathcal{M}_k^-). && (29)\end{aligned}$$

By Condition 2, with probability ζ_0,

$$c_1 \ge (\nu_1 + \nu_3)\|\nabla\mathcal{L}_k\| \tag{30}$$

and

$$c_2 \le \frac{4\kappa_H}{\nu_2} + \frac{L}{2} + \frac{\eta\kappa_H}{2}. \tag{31}$$

Therefore, for δ_k small enough to meet the conditions of the lemma,

$$\mathbb{P}(A_k \mid \mathcal{M}_k^-) \ge \zeta_0. \tag{32}$$

Now, suppose A_k holds. Hoeffding's inequality applies by Condition 4. Inequality 13 lets us cancel the remaining iteration-specific variables:

$$\begin{aligned} \mathbb{P}(\ell_k' \ge \eta m_k' \mid \mathcal{M}_k^+) &= 1 - \mathbb{P}(\mathcal{L}_k' - \ell_k' \ge \mathcal{L}_k' - \eta m_k' \mid \mathcal{M}_k^+) && (33)\\ &\ge 1 - \exp\left\{-\frac{(\mathcal{L}_k' - \eta m_k')^2 N_k}{2\sigma_k^2}\right\} && (34)\\ &\ge 1 - \exp\left\{-\frac{\nu_1^2\|\nabla\mathcal{L}_k\|^2\delta_k^2 N_k}{2\sigma_k^2}\right\} && (35)\\ &\ge \zeta_1. && (36)\end{aligned}$$

The lemma follows because

$$\mathbb{P}(\ell_k' \ge \eta m_k' \mid \mathcal{M}_k^-) \ge \mathbb{P}(A_k \mid \mathcal{M}_k^-)\mathbb{P}(\ell_k' \ge \eta m_k' \mid \mathcal{M}_k^+, A_k). \tag{37}$$

□

Lemma 3. *Define*

$$c_1 \triangleq [\nabla\mathcal{L}_k]^\intercal \frac{g_k}{\|g_k\|} - \eta\|g_k\| \quad \textit{and} \quad c_2 \triangleq 4\kappa_H \frac{\|\nabla\mathcal{L}_k\|}{\|g_k\|} + \frac{L}{2} + \frac{\eta\kappa_H}{2}. \tag{38}$$

If $\delta_k \le \delta_k^-$, then

$$\mathcal{L}_k' - \eta m_k' \ge c_1\delta_k - c_2\delta_k^2 \ \text{a.s.} \tag{39}$$

Proof. By Condition 1, for some $t \in (0,1)$,

$$\begin{aligned} \mathcal{L}_k' &= \mathcal{L}(\omega_k + s_k) - \mathcal{L}_k && (40)\\ &= s_k^\intercal \nabla\mathcal{L}_k + \frac{1}{2}s_k^\intercal \left[\nabla^2\mathcal{L}(\omega_k + ts_k)\right] s_k && (41)\\ &\ge s_k^\intercal\nabla\mathcal{L}_k - \frac{L}{2}\delta_k^2. && (42)\end{aligned}$$

To lower bound the first term, we first express the proposed step s_k in terms of g_k. Because s_k solves

$$\min_s g_k^\intercal s + \frac{1}{2}s^\intercal H_k s : \|s\| \le \delta_k, \tag{43}$$

there exists $\alpha_k \geq 0$ such that

$$(H_k + \alpha_k I)s_k = g_k. \tag{44}$$

The matrix $(H_k + \alpha_k I)$ is PSD. It follows that

$$\|(H_k + \alpha_k I)^{-1} g_k\| \leq \delta_k. \tag{45}$$

Therefore, by Condition 3,

$$\alpha_k \geq \frac{\|g_k\|}{\delta_k} - \kappa_H. \tag{46}$$

By Equality 44 and Inequality 46,

$$g_k^\intercal s_k = [(H_k + \alpha_k I)s_k]^\intercal s_k \tag{47}$$
$$= s^\intercal H_k s_k + \alpha_k s^\intercal s_k \tag{48}$$
$$\geq -\kappa_H \delta_k^2 + \alpha_k \delta_k^2 \tag{49}$$
$$\geq \|g_k\|\delta_k - 2\kappa_H \delta_k^2. \tag{50}$$

It follows that

$$s_k = \beta_k \frac{g_k}{\|g_k\|} + g^\perp, \tag{51}$$

for

$$\beta_k \geq \delta_k - \frac{2\kappa_H}{\|g_k\|}\delta_k^2. \tag{52}$$

for some $g^\perp \perp g_k$. For any $g^\perp$,

$$\|g^\perp\| \leq \frac{2\kappa_H}{\|g_k\|}\delta_k^2. \tag{53}$$

Now, with s_k expressed in terms of g_k, we lower bound the first term of Equation 42:

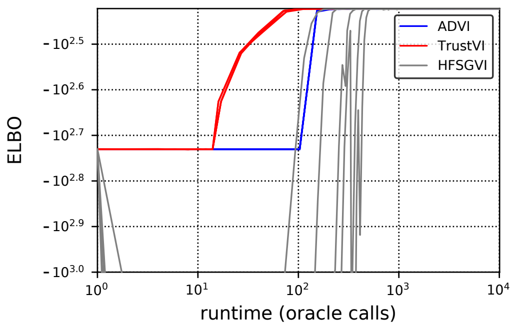
$$s_k^\intercal \nabla\mathcal{L}_k \geq \beta_k [\nabla\mathcal{L}_k]^\intercal \frac{g_k}{\|g_k\|} - \|\nabla\mathcal{L}_k\| \frac{2\kappa_H \delta_k^2}{\|g_k\|} \tag{54}$$
$$\geq \left[\delta_k - \frac{2\kappa_H}{\|g_k\|}\delta_k^2\right] [\nabla\mathcal{L}_k]^\intercal \frac{g_k}{\|g_k\|} - \|\nabla\mathcal{L}_k\| \frac{2\kappa_H \delta_k^2}{\|g_k\|} \tag{55}$$
$$\geq [\nabla\mathcal{L}_k]^\intercal \frac{g_k}{\|g_k\|}\delta_k - 4\kappa_H \frac{\|\nabla\mathcal{L}_k\|}{\|g_k\|}\delta_k^2 \tag{56}$$

Now, turning our attention to the improvement to the quadratic model:

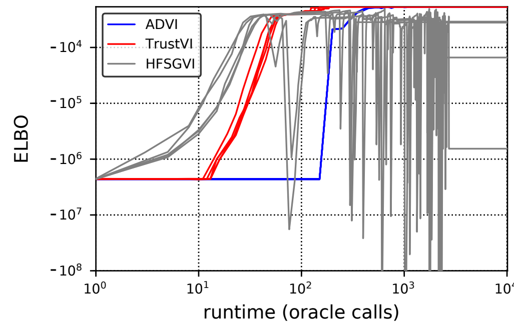
$$m_k' = g_k^\intercal + \frac{1}{2} s_k^\intercal H_k s_k \tag{57}$$
$$\leq \|g_k\|\delta_k + \frac{\kappa_H}{2}\delta_k^2. \tag{58}$$

The lemma follows from Inequality 42, Inequality 56, and Inequality 58. □

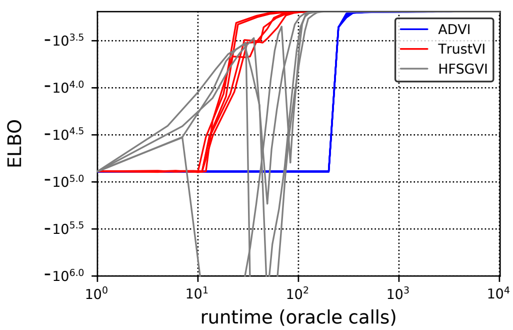
B Additional experiments



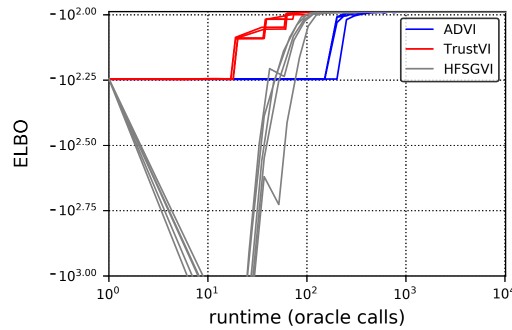
(a) Multinomial logistic regression ("Alligators") from [23]. 56-dimensional domain.



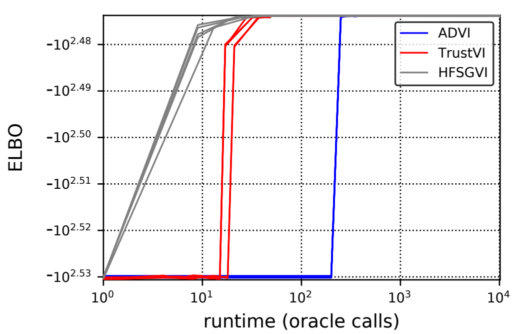
(b) A linear model with two predictors and interaction centered using conventional points ("Kid IQ interaction c2") from [20]. 10-dimensional domain.



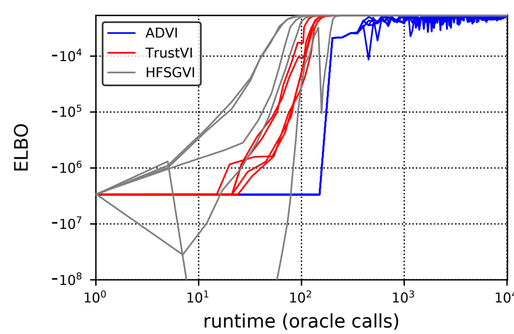
(c) A linear model with two predictors and a log log transformation ("Log Earn Log Height") from [20]. 8-dimensional domain.



(d) Random effect logistic regression ("Seeds") from [24]. 346-dimensional domain.



(e) Estimation of the Size of a Closed Population from Capture-Recapture Data ("Mt") from [25]. 8-dimensional domain.



(f) A linear model with two predictors and interaction ("Kid IQ interaction") from [20]. 10-dimensional domain.

Figure 2: Each panel shows optimization paths for five runs of ADVI, TrustVI, and HFSGVI, for a particular dataset and statistical model. Both axes are log scale.