Supplemental Material

Illustrations of the Forcing Operations.

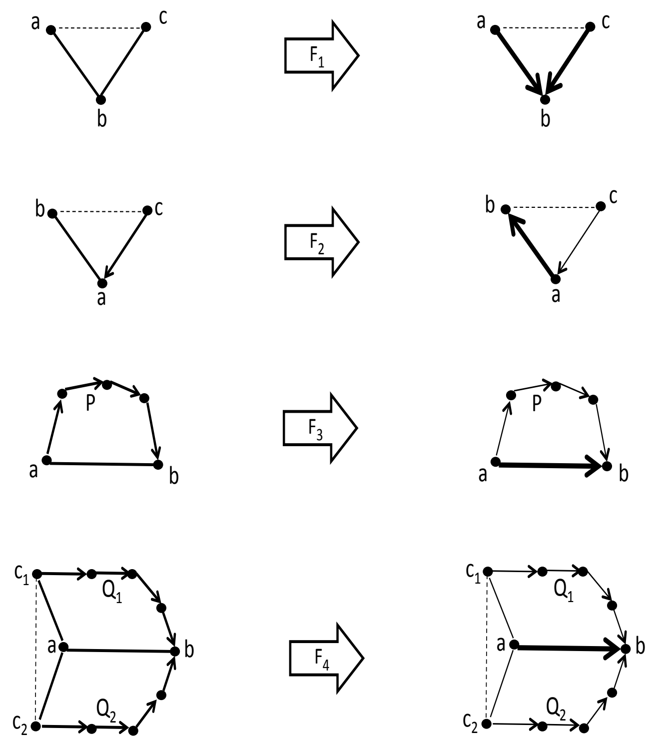

Figure 2: Forced Orientations.

Proof of Theorem 4.1.

Without loss of generality, we may assume the vertex ordering is $\{1, 2, \ldots, n\}$. Given this ordering, we want to know what is the probability that vertex t is the rth (smallest) vertex in a undirected clique in $\mathcal{U}$. Observe that there are $\binom{t-1}{r-1}$ possible r-cliques ending at vertex t. Take any such clique Q. Let A_Q be the indicator variable for Q to be a clique in $\mathcal{U}$.

To calculate the probability of this event, first note that each edge $e = (j, i), j < i \leq t$ in Q must have been selected in the random graph $C_{n,p}$. This occurs with probability $p^{\binom{r}{2}}$. Furthermore, no edge $(j, t) \in Q$ can be contained in any v-structure; otherwise its orientation will already have been discovered and it will not be in $\mathcal{U}$. For these events to occur, (j, t) must be chosen as an edge in the random graph **and** for all $k < t, k \notin Q$, either (k, t) is not an edge or (k, t) is an edge and (k, q) is an edge for all $q \in Q, q < t$. Thus, if t is the rth smallest vertex in Q,

$$P(A_Q) \quad \leq \quad p^{\binom{r}{2}} \cdot \left((1-p) + p \cdot p^{r-1}\right)^{t-r}$$

Therefore, by the union bound, the probability that any r-clique ends at vertex t is at most

$$\binom{t-1}{r-1} \cdot p^{\binom{r}{2}} \cdot (1-p+p^r)^{t-r} \quad \leq \quad \binom{t}{r} \cdot p^{\binom{r}{2}} \cdot (1-p+p^r)^{t-r}$$