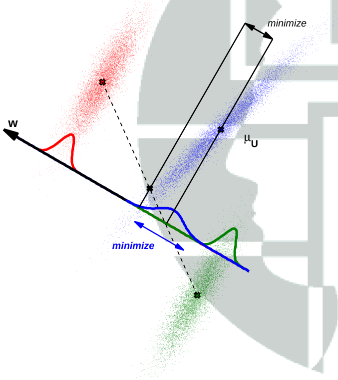
# W17: An Analysis of Inference with the Universum

Fabian Sinz, Olivier Chapelle, Alekh Agarwal & Bernhard Schölkopf

Data-dependent regularization with the Universum $\mathfrak{U} = \{\boldsymbol{z}_1, \ldots, z_q\}$ :

$$\min_{\boldsymbol{w},b} \quad \frac{1}{2}||\boldsymbol{w}||^2 + C_{\mathfrak{L}} \sum_{i=1}^{m} H_{+1,-1}[\langle \boldsymbol{w}, \boldsymbol{x}_i \rangle + b] + C_{\mathfrak{U}} \sum_{j=1}^{q} I[\, \langle \boldsymbol{w}, \boldsymbol{z}_j \rangle + b\,]$$

with $H =$  and $I =$  ($\mathfrak{U}$-SVM) or $H =$  and $I =$  ($\mathfrak{U}_{\mathfrak{ls}}$-SVM)




## Results for $\mathfrak{U}$-SVM

- For $C_{\mathfrak{U}} = \infty$ the solution $\boldsymbol{w}^*$ lives on the orthogonal complement of the affine subspacespanned by $\mathfrak{U}$
- For $C_{\mathfrak{U}} < \infty$ the $\mathfrak{U}$-SVM minimizes an upper bound on the "$L_1$-variance"
  $C_{\mathfrak{U}} \sum_{j=1}^{q} |\langle \boldsymbol{w}^*, \boldsymbol{z}_j \rangle + b^*| \geq C_{\mathfrak{U}} \min_b \sum_{j=1}^{q} |\langle \boldsymbol{w}^*, \boldsymbol{z}_j \rangle + b|$

## Results for $\mathfrak{U}_{\mathfrak{ls}}$-SVM

- The $\mathfrak{U}_{\mathfrak{ls}}$-SVM is equivalent (up to a linear constraint) to a hybird of *oriented PCA* and *Fisher Disriminant Analysis* where $\mathfrak{U}$ specifies the "noise directions" in oriented PCA

## Results

- Both, the *absolute position* as well as the *main principal axes* influence the solution $\boldsymbol{w}^*$

The Universum set $\mathfrak{U}$ allows to specify implicit invariance directions but must be carefully chosen